STABILIZABILITY OF BIBO INTEGRO-DIFFERENTIAL SYSTEMS WITH TWO DISTRIBUTED DELAYS

Carlos F. ALASTRUEY

Department of Electricity and Electronics
University of the Basque Country
P.O. Box 644 – 48080 Bilbao, Spain

Juan M. SANDOVAL

Department of Automatic Control
Electronics and Systems Engineering
Public University of Navarra
Campus de Arrosadia s/n
31006 Pamplona, Spain

Abstract. Control laws' design strategies are developed to stabilize a class of BIBO integro-differential systems with two distributed delays by using an extended system.

Key words: integro-differential systems, distributed delays, stabilizability.

1. Introduction and problem statement. Mathematical models with delays are utilized to deal with a wide class of physical systems. For large delay-differential systems, an approximate method of solution has been developed by Alastruey and González de Mendivil (1994). The stabilizability conditions for systems with general delays in state were extended by Pandolfi (1975) and also by Bhat and Koivo (1976). In a work due to Olbrot (1978) open-loop stabilizability problems for systems with control and state delays were defined. Conditions for the delay-independent stabilization of linear systems were given by Amemiya et al. (1986), being the upper bound or the lower bound of the decay rates assignable, and Akazawa et al. (1987), by using in the proof matrices with some of their elements being arbitrary. In addition, Fiagbedzi and Pearson (1986, 1990) introduced techniques for the feedback and output feedback stabilization of delay systems by using a generalization of the transformation
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method. Furthermore, Mori et al. (1983) developed a way to stabilize linear systems with delayed state.

The stability of a linear delay-differential system with a point delay in its state has been studied in different works (Mori et al., 1992; Hmamed, 1985, 1986 a-b; Mori, 1986; Bourlès, 1987 and Mori and Kokame, 1989). In particular, the problem of stability for a scalar differential system with two point delays in its state has been considered by several authors (see, for instance, Jury and Mansour, 1982). However, the exact delay-dependent algebraic stability conditions of such a system were lately provided by Schoen and Geering (1993); such conditions were obtained by using an instability criterion together with the D-decomposition method. In a recent paper (Aistrauey et al., 1994), several criteria in order to design stabilizing control laws for integro-differential systems with two distributed delays in their state were introduced by using an associated extended system under the form of a linear-differential system with two point delays in its state. The method utilized to investigate the stabilizability of a system subject to two bounded known distributed delays was applicable to diagonally dominant systems and was based on testing scalar constraints for the various matrices defining the system. In this paper we particularize the above-mentioned results for the case of a class of BIBO (bi-input bi-output) integro-differential systems with two distributed delays in their state.

The paper is organized as follows: Section 2 introduces an important result on the stability for a system with two point delays in its state that will be used in the sequel. Section 3 shows the main stabilizability result for a class of BIBO systems with two distributed delays in their state. Finally, conclusions end the paper.

2. Stability for a BIBO system with two point delays. The following results will be useful in the next section in order to deduce stabilizability conditions for a BIBO integro-differential system with two distributed delays in its state given in terms of algebraic relations.

**Theorem 2.1** (Aistrauey et al., 1994). Consider the following linear MIMO system with two point delays in its vector-state

\[ \dot{x}(t) = A_0x(t) + A_1x(t - h) + A_2x(t - 2h) \]  

with initial condition \( \varphi_x : [-2h, 0] \rightarrow \mathbb{R}^n; \varphi_x(0) = x(0) \) being absolutely continuous with possible bounded discontinuities on a subset of \([-2h, 0]\) of
zero measure, and where $A_0, A_1$ and $A_2$ are real constant $n \times n$-matrices and $h > 0$. Consider the following $n \times n$-matrices

$$A_k = A_{kd} + \overline{A}_{kd}, \quad k = 0, 1, 2,$$

(2)

$$D = \text{diag} \left[ -\left( a_{ii}^2 \right) - \frac{\pi}{2h} \right],$$

(3)

$$E = -\left( A_{0d} + A_{1d} + A_{2d} \right),$$

(4)

$$F = \text{diag} \left[ \frac{y_i \cos(y_i h)}{\sin(y_i h)} \right],$$

(5)

$$G = \text{diag} \left[ \frac{y_i}{\sin(y_i h)} + 2a_{ii}^2 \cos(y_i h) \right],$$

(6)

where $A_0, A_1, A_2$ are decomposed according to (2) where $A_{kd} = \text{diag} (a_{ii}^k)$, $(k = 0, 1, 2)$ are diagonal matrices, and $\overline{A}_{kd} = (a_{ij}^k, i \neq j, \text{otherwise} \, 0) = (\overline{a}_{ij}^k)$ are matrices with zero entries in their main diagonal; $a_{ii}^2$ are the elements of the diagonal matrix $A_{2d}$ and $y_i \in [0, \frac{\pi}{h}]$, $(i = 1, \ldots, n)$ are real numbers. Then the time-delay system (1) with $D$ positive is asymptotically stable if the following four conditions hold for some set of values $y_i \in [0, \frac{\pi}{h}]$, $(i = 1, \ldots, n)$

(i) $E$ is positive

(ii) $A_{0d} = F + A_{2d}$

(iii) $A_{1d} + G$ is positive,

(iv)

$$\sum_{k=0}^{2} \left( \sum_{i=1}^{n} \left| a_{ij}^k \right|^2 \right)^{1/2}$$

$$< \text{Max} \left\{ \left( \sum_{i=1}^{n} \left| a_{ii}^0 \right|^2 \right)^{1/2} - \left( \sum_{i=1}^{n} \left| a_{ii}^1 \right|^2 \right)^{1/2} - \left( \sum_{i=1}^{n} \left| a_{ii}^2 \right|^2 \right)^{1/2} \right\}.$$

(10)
Corollary 2.1. Consider the following linear BIBO system with two point delays in its vector-state

\[ \dot{x}(t) = A_0 x(t) + A_1 x(t - h) + A_2 x(t - 2h) \]  

(11)

with initial condition \( \varphi_x : [-2h, 0] \to R^n \varphi_x(0) = x(0) \) being absolutely continuous with possible bounded discontinuities on a subset of \([-2h, 0]\) of zero measure, and where \( A_0, A_1 \) and \( A_2 \) are real constant \( n \times n \)-matrices and \( h > 0 \). Consider the following 2 \( \times \) 2-matrices

\[ A_k = A_{kd} + \bar{A}_{kd}, \quad k = 0, 1, 2, \]  

(12)

\[ D = \text{diag} \left[ -\left( |a_{ii}^2| - \frac{\pi}{2h} \right) \right], \]  

(13)

\[ E = -(A_{0d} + A_{1d} + A_{2d}), \]  

(14)

\[ F = \text{diag} \left[ \frac{y_i \cos(y_i h)}{\sin(y_i h)} \right], \]  

(15)

\[ G = \text{diag} \left[ \frac{y_i \sin(y_i h)}{\sin(y_i h)} + 2a_{ii}^2 \cos(y_i h) \right], \]  

(16)

where \( A_0, A_1, A_2 \) are decomposed according to (12) where \( A_{kd} = \text{diag}(a_{ii}^{kd}), \quad (k = 0, 1, 2) \) are diagonal matrices, and \( \bar{A}_{kd} = (a_{ij}^{kd}, \quad i \neq j, \quad \text{otherwise} \quad 0) = (\bar{a}_{ij}^{kd}) = A_k = A_{kd} \) are matrices with zero entries in their main diagonal; \( a_{ii}^{kd} \) are the elements of the diagonal matrix \( A_{kd} \) and \( y_i \in [0, \frac{\pi}{h}], \quad (i = 1, \ldots, n) \) are real numbers. Then the time-delay system (11) with \( D \) positive is asymptotically stable if the conditions (7)-(9) for matrices (12)-(16) hold for some set of values \( y_i \in [0, \frac{\pi}{h}], \quad (i = 1, 2) \) as well as the following condition:

\[
\sum_{k=0}^{2} \left( \sum_{i,j \neq k} |a_{ij}^{kd}|^2 \right)^{1/2} < \text{Max} \left\{ \left( \sum_{i=1}^{2} |a_{ii}^{0}|^2 \right)^{1/2} - \left( \sum_{i=1}^{2} |a_{i1}^{1}|^2 \right)^{1/2} - \left( \sum_{i=1}^{2} |a_{i2}^{2}|^2 \right)^{1/2}, \right. \\
\left. \left( \sum_{i=1}^{2} |a_{ii}^{1}|^2 \right)^{1/2} - \left( \sum_{i=1}^{2} |a_{i1}^{0}|^2 \right)^{1/2} - \left( \sum_{i=1}^{2} |a_{i2}^{2}|^2 \right)^{1/2}, \right. \\
\left. \left( \sum_{i=1}^{2} |a_{ii}^{2}|^2 \right)^{1/2} - \left( \sum_{i=1}^{2} |a_{i1}^{0}|^2 \right)^{1/2} - \left( \sum_{i=1}^{2} |a_{i1}^{2}|^2 \right)^{1/2} \right\} \]  

(17)
Proof. The proof follows immediately by applying Theorem 2.1 to system (11).

3. Main stabilizability result for a BIBO system with two distributed delays. In this section conditions for a control law to stabilize a BIBO integro-differential system with two distributed delays in its state will be discussed by using an associated extended system. A main result is introduced.

Consider the following BIBO integro-differential system with two distributed delays in its state, and containing a control law with two point delays and a control law with two distributed delays.

\[ \dot{x}(t) = A_0 x(t) + \int_0^t A_1 x(t' - h) dt' \int_0^t A_2 x(t' - 2h) dt' + B_{01}^1 u_1(t) + B_{01}^1 u_1(t - h) + B_{11}^1 u_1(t - 2h) + \int_0^t \left[ B_{02}^{21} u_2(t') + B_{12}^{21} u_2(t' - h) + B_{22}^{21} u_2(t' - 2h) \right] dt' \]  

(18)

with \( x(t) = g(t) \) for all \( t < 0 \), where \( A_0, A_1, A_2, B_{01}^1, B_{01}^1, B_{11}^1, B_{02}^{21}, B_{12}^{21}, B_{22}^{21}, B_{02}^{22}, B_{12}^{22}, B_{22}^{22} \in W \subset \mathbb{R}^{2 \times 2}, u_1(t), u_2(t) = 0 \ \forall t < 0 \) and \( h \in \mathbb{R}^+ \).

**Theorem 3.1 (Asymptotic stabilizability).** Consider two control laws \( u_1(t), u_2(t) \) being continuous and differentiable on \([0, \infty)\) defined by the delay-differential equations

\[
\begin{bmatrix}
\dot{u}_1(t) \\
\dot{u}_2(t)
\end{bmatrix} = \begin{bmatrix}
D_{01}^{11} & 0 \\
0 & D_{02}^{22}
\end{bmatrix} \begin{bmatrix}
x(t) \\
\dot{x}(t)
\end{bmatrix} + \begin{bmatrix}
E_{11}^{11} & 0 \\
0 & E_{12}^{22}
\end{bmatrix} \begin{bmatrix}
\dot{u}_1(t) \\
\dot{u}_2(t)
\end{bmatrix} + \begin{bmatrix}
D_{01}^{11} & 0 \\
0 & D_{02}^{22}
\end{bmatrix} \begin{bmatrix}
x(t - h) \\
\dot{x}(t - h)
\end{bmatrix} + \begin{bmatrix}
E_{11}^{11} & 0 \\
0 & E_{12}^{22}
\end{bmatrix} \begin{bmatrix}
\dot{u}_1(t - h) \\
\dot{u}_2(t - h)
\end{bmatrix} + \begin{bmatrix}
D_{01}^{11} & 0 \\
0 & D_{02}^{22}
\end{bmatrix} \begin{bmatrix}
x(t - 2h) \\
\dot{x}(t - 2h)
\end{bmatrix} + \begin{bmatrix}
E_{11}^{11} & 0 \\
0 & E_{12}^{22}
\end{bmatrix} \begin{bmatrix}
\dot{u}_1(t - 2h) \\
\dot{u}_2(t - 2h)
\end{bmatrix},
\]

and subject to initializations \( u_i(t) = \varphi_{iu}(t) \) on \([-2h, 0]\) with \( \varphi_{iu}(t) \) being absolutely continuous with perhaps bounded discontinuities on some subsets \( i_i \subset [-2h, 0] \) of zero measures; \( i = 1, 2 \).

Define the following \( 4 \times 4 \)-matrices
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\[ \begin{align*}
\tilde{A}_0 & \equiv \begin{bmatrix} 0 & I_n \\ 0 & A_0 \end{bmatrix} ; \\
\tilde{A}_1 & \equiv \begin{bmatrix} 0 & 0 \\ A_1 & 0 \end{bmatrix} ; \\
\tilde{A}_2 & \equiv \begin{bmatrix} 0 & 0 \\ A_2 & 0 \end{bmatrix} ; \\
\tilde{B}_0 & \equiv \begin{bmatrix} B_{11}^0 & 0 \\ 0 & B_{22}^0 \end{bmatrix} ; \\
\tilde{B}_1 & \equiv \begin{bmatrix} B_{11}^1 & 0 \\ 0 & B_{22}^1 \end{bmatrix} ; \\
\tilde{B}_2 & \equiv \begin{bmatrix} B_{11}^2 & 0 \\ 0 & B_{22}^2 \end{bmatrix} ;
\end{align*} \tag{20} \]

\[ \begin{align*}
\tilde{D}_0 & \equiv \begin{bmatrix} D_{11}^0 & 0 \\ 0 & D_{22}^0 \end{bmatrix} ; \\
\tilde{D}_1 & \equiv \begin{bmatrix} D_{11}^1 & 0 \\ 0 & D_{22}^1 \end{bmatrix} ; \\
\tilde{D}_2 & \equiv \begin{bmatrix} D_{11}^2 & 0 \\ 0 & D_{22}^2 \end{bmatrix} ;
\end{align*} \tag{21} \]

\[ \begin{align*}
\tilde{E}_0 & \equiv \begin{bmatrix} E_{11}^0 & 0 \\ 0 & E_{22}^0 \end{bmatrix} ; \\
\tilde{E}_1 & \equiv \begin{bmatrix} E_{11}^1 & 0 \\ 0 & E_{22}^1 \end{bmatrix} ; \\
\tilde{E}_2 & \equiv \begin{bmatrix} E_{11}^2 & 0 \\ 0 & E_{22}^2 \end{bmatrix} .
\end{align*} \tag{22} \]

Define the following $8 \times 8$-matrices

\[ \begin{align*}
\tilde{A}_0 & \equiv \begin{bmatrix} \tilde{A}_0 & \tilde{B}_0 \\ \tilde{D}_0 & \tilde{E}_0 \end{bmatrix} ; \\
\tilde{A}_1 & \equiv \begin{bmatrix} \tilde{A}_1 & \tilde{B}_1 \\ \tilde{D}_1 & \tilde{E}_1 \end{bmatrix} ; \\
\tilde{A}_2 & \equiv \begin{bmatrix} \tilde{A}_2 & \tilde{B}_2 \\ \tilde{D}_2 & \tilde{E}_2 \end{bmatrix} ;
\end{align*} \tag{24} \]

\[ \tilde{D} \equiv \text{diag} \left[ -\left( |\tilde{a}_{ij}^{2d}| - \frac{\pi}{h} \right) \right] \quad \text{(see Eq. 29 below)}, \tag{25} \]

where $\tilde{a}_{ij}^{2d}$ are the elements of $\tilde{A}_{2d}$

\[ \tilde{E} \equiv -\left( \tilde{A}_{0d} + \tilde{A}_{1d} + \tilde{A}_{2d} \right) \quad \text{(see Eq. 29 below)} ; \tag{26} \]

\[ \tilde{F} \equiv \text{diag} \left[ \frac{y_i \cos(y_i h)}{\sin(y_i h)} \right] ; \tag{27} \]

\[ \tilde{G} \equiv \text{diag} \left[ \frac{y_i}{\sin(y_i h)} + 2\tilde{a}_{ii}^{2d} \cos(y_i h) \right] \quad \text{(see Eq. 29 below)}, \tag{28} \]

where $y_i \in [0, \frac{\pi}{h}]$, ($i = 1, \ldots , 8$), are real numbers.

Decompose

\[ \tilde{A}_k = \tilde{A}_{kd} + \overline{\tilde{A}}_{kd} \quad (k = 0, 1, 2), \tag{29} \]

where $\tilde{A}_{kd}$ and $\overline{\tilde{A}}_{kd}$ are the diagonal and nondiagonal submatrices defined similarly as for (1).

Control laws (19) provide asymptotic stability for system (18) if the following five conditions hold for some set of values $y_i \in [0, \frac{\pi}{h}]$, ($i = 1, \ldots , 8$)
(i) \( \hat{D} \) is positive;  
(ii) \( \hat{E} \) is positive;  
(iii) \( A_{0d} = \hat{F} + \hat{A}_{2d} \);  
(iv) \( \hat{A}_{1d} + \hat{G} \) is positive;  
(v)

\[
\sum_{k=0}^{2} \left( \sum_{i,j=1}^{8} |\hat{a}_{ij}|^2 \right)^{1/2} < \text{Max} \left\{ \left( \sum_{i=1}^{8} |\hat{a}_{i0}|^2 \right)^{1/2} - \left( \sum_{i=1}^{8} |\hat{a}_{i1}|^2 \right)^{1/2} - \left( \sum_{i=1}^{8} |\hat{a}_{i2}|^2 \right)^{1/2} \right\} \]  

Proof. Consider the free part of system (18)

\[
\dot{x}(t) = A_0 x(t) + \int_0^t A_1 x(t' - h) \, dt' + \int_0^t A_2 x(t' - 2h) \, dt' 
\]

with \( x(t) = g(t) \) for all \( t < 0 \). By differentiating system (35) one gets

\[
\ddot{x}(t) = A_0 \dot{x}(t) + A_1 x(t - h) + A_2 x(t - 2h) 
\]

with \( \dot{z}(t) = \dot{g}(t) \) \( \forall t < 0 \). Define the 4-vector \( \ddot{z}(t) \equiv \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \). From equations (35) and (36) one gets

\[
\ddot{z}(t) = \begin{bmatrix} \dot{z}(t) \\ \ddot{z}(t) \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ \hat{A}_0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_1 & 0 \end{bmatrix} \begin{bmatrix} x(t-h) \\ \dot{x}(t-h) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_2 & 0 \end{bmatrix} \begin{bmatrix} x(t-2h) \\ \dot{x}(t-2h) \end{bmatrix} 
\]
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By using definition in equation (20), Eq. 37 can be rewritten as follows

\[ \dot{z}(t) = \tilde{A}_0 \tilde{z}(t) + \tilde{A}_1 \tilde{z}(t-h) + \tilde{A}_2 \tilde{z}(t-2h). \] (38)

System (38) is called "associated extended system" of the free system (35). By applying the proposed control laws to system (38) one gets

\[ \dot{\tilde{z}}(t) = \tilde{A}_0 \tilde{z}(t) + \tilde{A}_1 \tilde{z}(t-h) + \tilde{A}_2 \tilde{z}(t-2h) \]
\[ + \tilde{B}_0 \tilde{u}(t) + \tilde{B}_1 \tilde{u}(t-h) + \tilde{B}_2 \tilde{u}(t-2h), \] (39)

where

\[ \tilde{u}(t) \equiv \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \] (40)

Observe that Eqs. (19) and (39) can be rewritten in one single delay-differential equation as follows

\[ \begin{bmatrix} \dot{\tilde{z}}(t) \\ \dot{\tilde{u}}(t) \end{bmatrix} = \begin{bmatrix} \tilde{A}_0 & \tilde{B}_0 \\ \tilde{D}_0 & \tilde{E}_0 \end{bmatrix} \begin{bmatrix} \tilde{z}(t) \\ \tilde{u}(t) \end{bmatrix} + \begin{bmatrix} \tilde{A}_1 & \tilde{B}_1 \\ \tilde{D}_1 & \tilde{E}_1 \end{bmatrix} \begin{bmatrix} \tilde{z}(t-h) \\ \tilde{u}(t-h) \end{bmatrix} \]
\[ + \begin{bmatrix} \tilde{A}_2 & \tilde{B}_2 \\ \tilde{D}_2 & \tilde{E}_2 \end{bmatrix} \begin{bmatrix} \tilde{z}(t-2h) \\ \tilde{u}(t-2h) \end{bmatrix}. \] (41)

Define

\[ z(t) = [\tilde{z}^T(t) : \tilde{u}^T(t)]^T, \] (42)

where \( z(t) \) is an 8-vector and \( \tilde{A}_0, \tilde{A}_1, \tilde{A}_2 \) are \( 8 \times 8 \) matrices defined in (24). Then, equation (41) can be rewritten as

\[ \dot{z}(t) = \tilde{A}_0 z(t) + \tilde{A}_1 z(t-h) + \tilde{A}_2 z(t-2h). \] (43)

If, by hypothesis, conditions (30)-(33) hold, then by Corollary 2.1 system (43) is asymptotically stable, i.e., system (38) is asymptotically stabilizable by control law (40). Observe that the controlled associated extended system (39) becomes

\[ \dot{\tilde{z}}(t) = A_0 x(t) + \int_0^t A_1 x(t' - h) dt' \int_0^t A_2 x(t' - 2h) dt' \]
\[ + B_0^1 u_1(t) + B_1^1 u_1(t - h) + B_2^1 u_1(t - 2h) \] (44)
with initial conditions \( x(\tau) = \varphi_{1x}(\tau); \ \tau \in [2h, 0); \ x(0) = \varphi_{1x}(0); \)

\[
\ddot{x}(t) = A_0 \dot{x}(t) + A_1 x(t - h) + A_2 x(t - 2h) + B_0^{22} u_2(t) + B_1^{22} u_2(t - h) + B_2^{22} u_2(t - 2h). \tag{45}
\]

The integration of Eq. 45 yields

\[
\dot{x}(t) = A_0 x(t) + \int_0^t A_1 x(t' - h) \, dt' + \int_0^t A_2 x(t' - 2h) \, dt' + \int_0^t B_0^{22} u_2(t') \, dt' \\
+ \int_0^t B_1^{22} u_2(t' - h) \, dt' + \int_0^t B_2^{22} u_2(t' - 2h) \, dt' \tag{46}
\]

with initial conditions \( x(\tau) = \varphi_{2x}(\tau); \ \tau \in [2h, 0); \ x(0) = \varphi_{2x}(0). \)

Finally, the application of the superposition principle to the solution (44) and (46) yields

\[
\dot{x}(t) = A_0 x(t) + \int_0^t A_1 x(t' - h) \, dt' + \int_0^t A_2 x(t' - 2h) \, dt' \\
+ \int_0^t B_1^{11} u_1(t) + B_2^{11} u_1(t - h) + B_2^{11} u_2(t - 2h) \\
+ \int_0^t [B_0^{22} u_2(t') + B_1^{22} u_2(t' - h) + B_2^{22} u_2(t' - 2h)] \, dt' \tag{47}
\]

with initial conditions \( x(\tau) = \varphi_{1x}(\tau) + \varphi_{2x}(\tau) \) and the controlled system (47) – which coincides with (18) – is asymptotically stable, since the stability property is independent from the particular initial values \( \varphi_{ix}(\tau) \) provided that for any vector function norm, \( ||\varphi_{ix}||_{[-2h, 0]} < \infty \) \( (i = 1, 2). \)

Theorem 3.1 provides a way to evaluate the asymptotic stabilizability of a BIBO delay-differential system with two distributed delays in its state. The evaluation is made on the context of a set of algebraic relations, which are very suitable for computer applications.

4. Conclusions. This paper provides sufficient conditions for testing asymptotic stabilizability of a class of BIBO delay-differential systems with distributed delays in its state.
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Conditions are given in terms of algebraic relations, useful for computer applications. The control is given in terms of two independent control laws defined by using dynamic differential equations which contain point delays. The control laws are independent in the sense that each one of them are applied independently to state equation (i.e., they are not interconnected).

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C. F. Alastruey was born in the Basque Country in 1963. In 1981 he became Bahá'í. He received the degree of Physicist (area: Electronics and Control) from the University of the Basque Country in 1988, and the degree of Doctor in Physics from the same university in 1993. He is professor in the Department of Electricity and Electronics in the University of the Basque Country. His research interests are mathematical and computing modelling of functional differential equations, control and identification of delay-differential systems and control of industrial processes.

Juan M. Sandoval obtained the degree of physicist Universidad Complutense of Madrid (Spain) in 1962, and the PhD by the same university in 1972. After having been Professor in several universities in Madrid, San Sebastian and the University of the Basque Country, he became a Professor with the Department of automatic Control, Electronic and Systems Engineering in the Public University of Navarra (Spain). Currently he is the Head of such a Department. His research interests include controllers, optimal control and digital filtering.