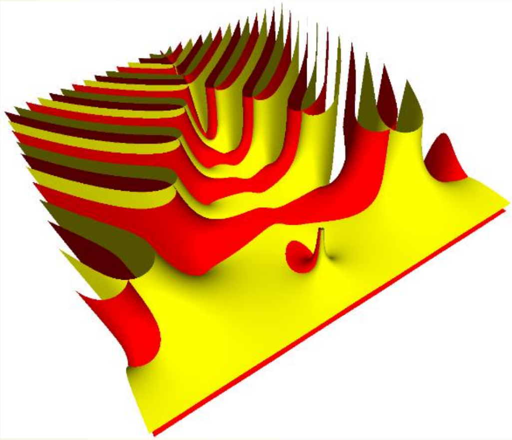
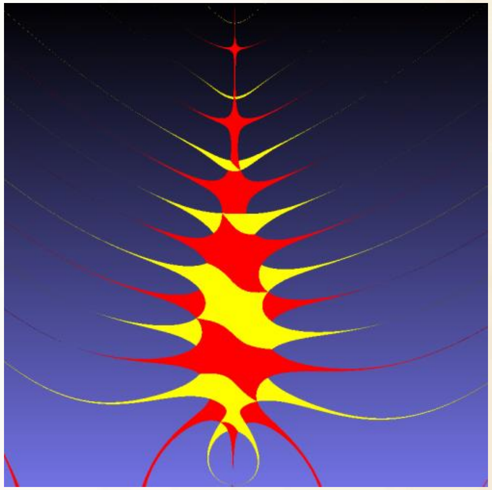
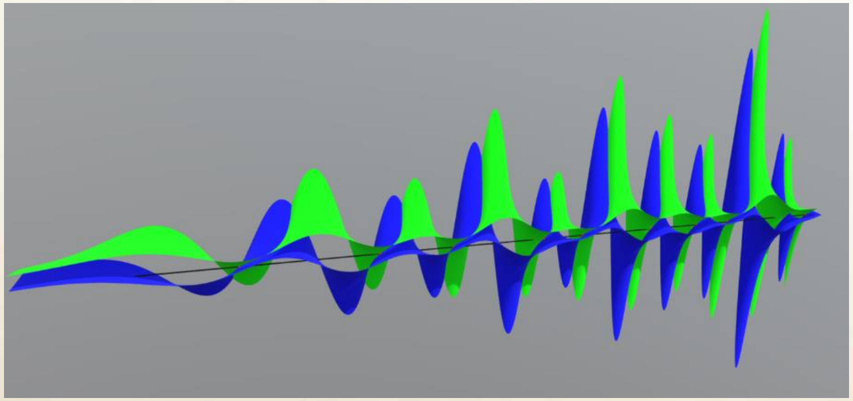
Apie Rymano dzeta funkcijos 3D vizualizavimo algoritmus ir priemones

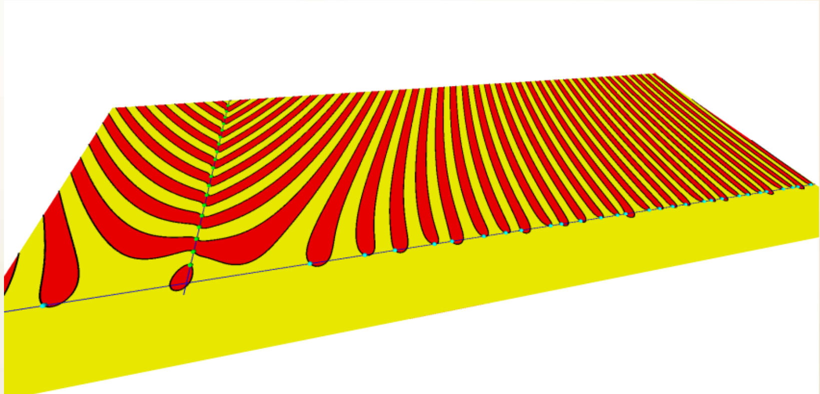




Martynas Sabaliauskas

2019-03-27





Kodėl 3D vizualizacija?

$$S = S(x(u,v), y(u,v), z(u,v)),\ u \in [0, 2\pi),\ v \in [0, \pi]:$$

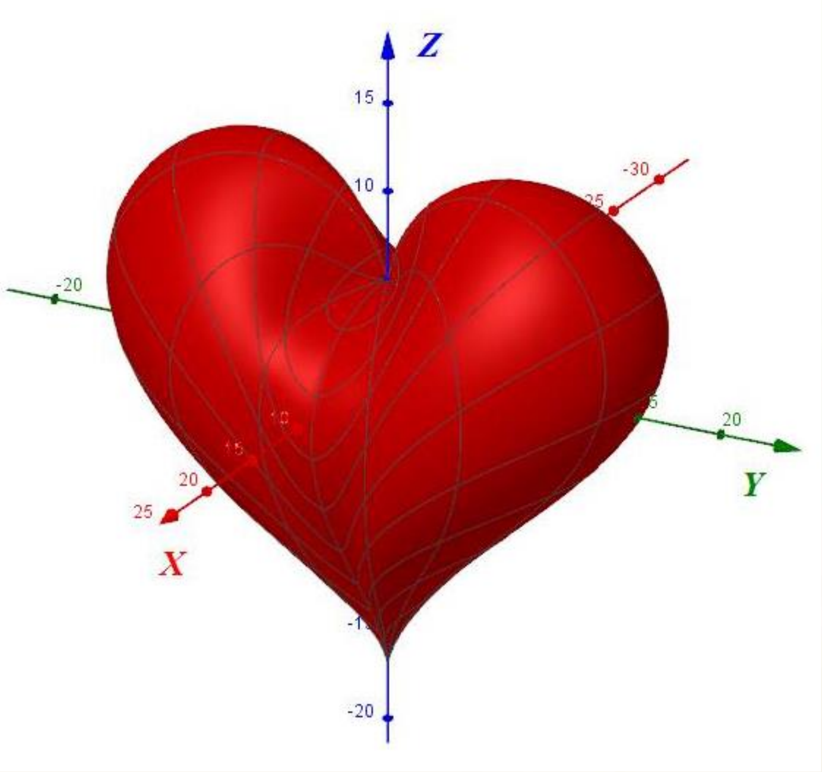
$$x(u,v) = \left(\sqrt{\sin^3(v)}\,\left(4\sin(u)\right)^2 - \frac{11\cos^2(v)\left(7\sqrt{11} - 17\right)\,(\cos(u)+1)}{\left(6\sqrt{11} - 11\right)\,(\cos(u)+1) - 25} \right)\cos(v)\,\sin(u),$$

$$y(u,v) = \left(\left(4\sin(v)\,\sin(u)\right)^2 - \frac{11\left(1 - \sqrt{\sin^3(v)}\right)\left(7\sqrt{11} - 17\right)\,(\cos(u)+1)}{\left(6\sqrt{11} - 11\right)\,(\cos(u)+1) - 25} \right)\sin(v)\,\sin(u),$$

$$z(u,v) = \sqrt{\sin^3(v)}\,\left(13\cos(u) - 5\cos(2u) - 2\cos(3u) - \cos(4u)\right) +$$

$$+\ \frac{\left(1 - \sqrt{\sin^3(v)}\right)\left(\left(7\sqrt{11} - 17\right)\,(\cos(u)+1)\left(55\cos(u) - 187 - 72\sqrt{11}\right) + 4250\right)}{10\left(\left(6\sqrt{11} - 11\right)\,(\cos(u)+1) - 25\right)}$$

Atsakymas



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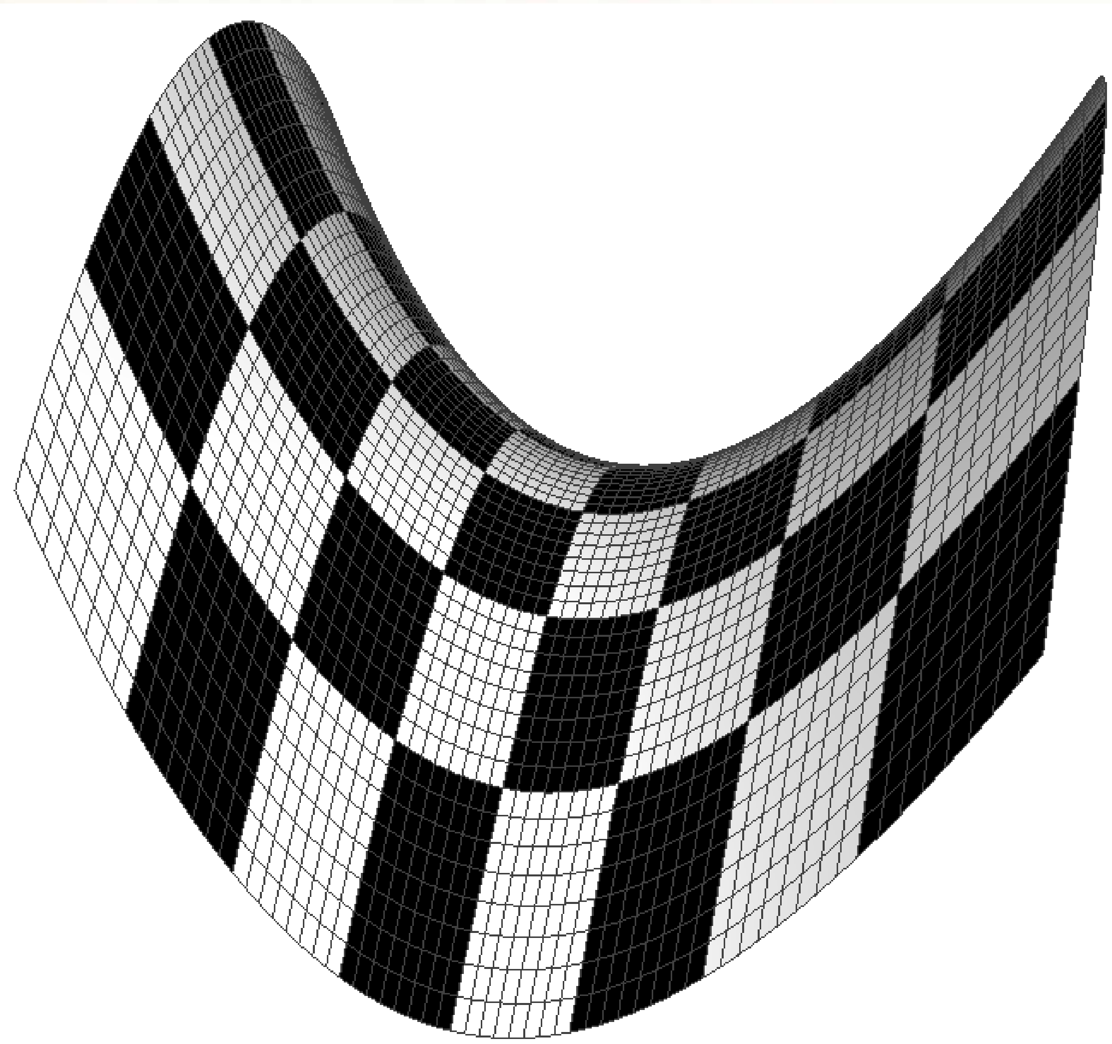

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4 6 7 72 71 0 0 0
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4 8 9 74 73 255 255 255

      <...>
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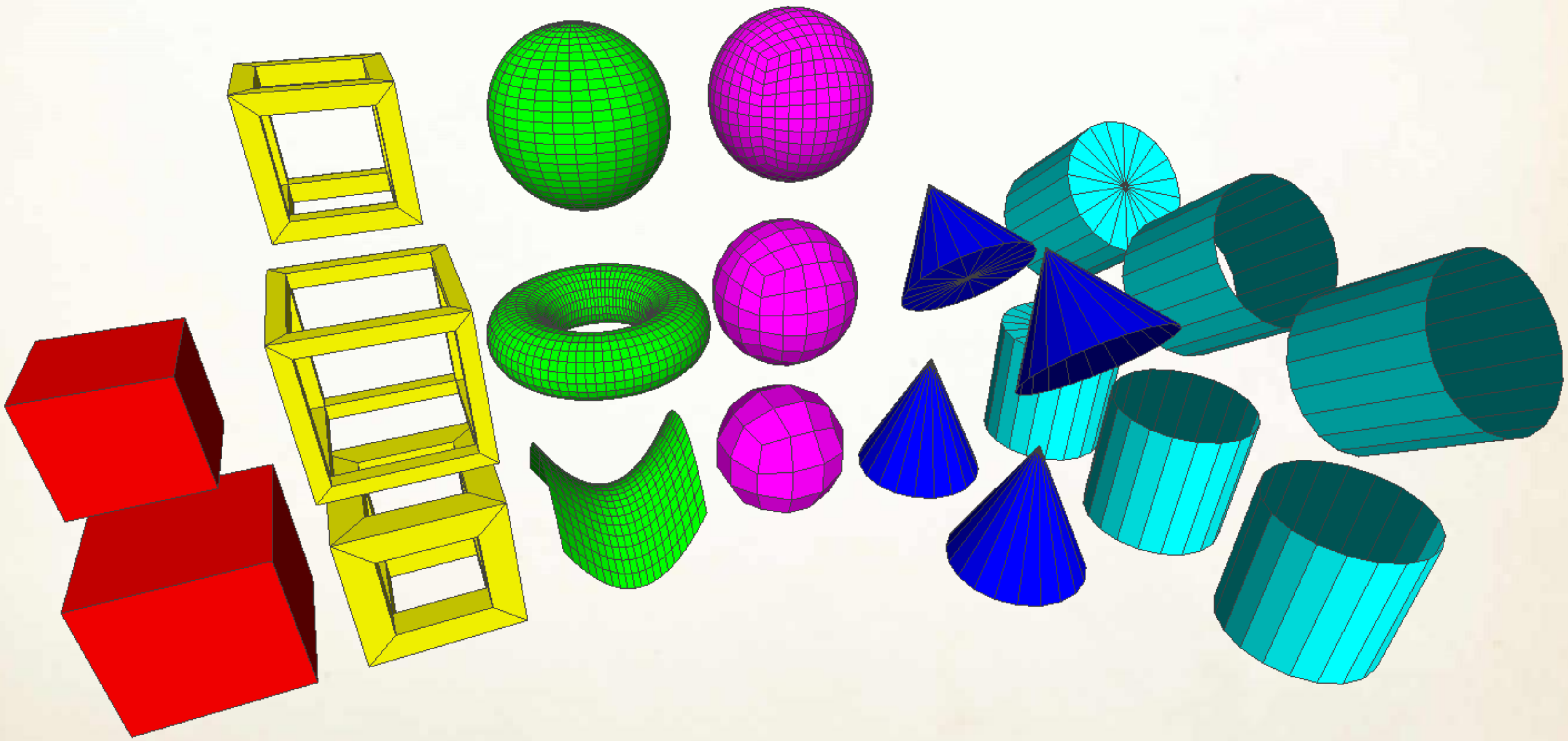
*Modulis **add.py** ir galimos detalės*



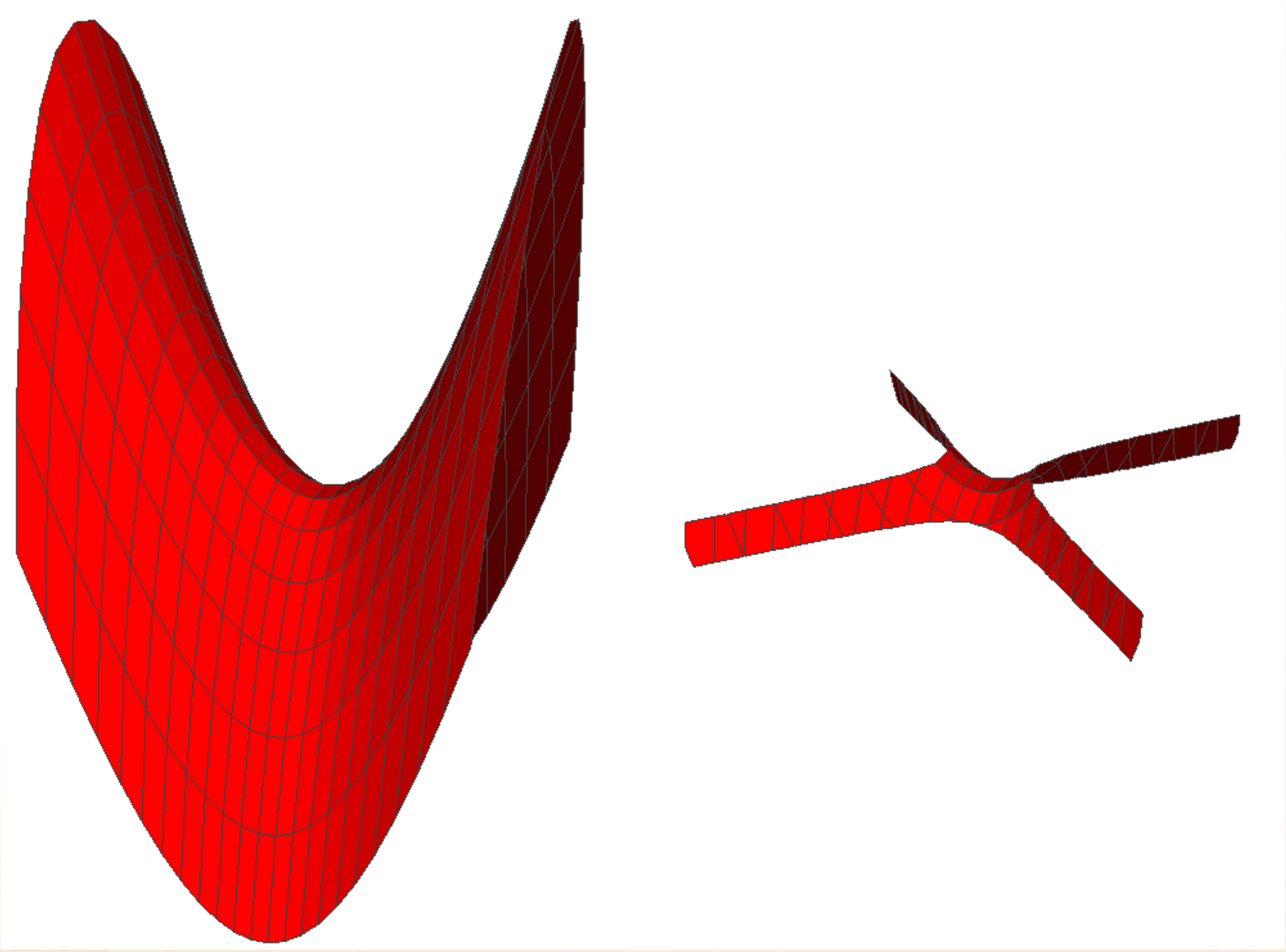
Naujos funkcijos 3D modeliams kurti

- add.py modulis:
  - $z = f(x,y)$ atvaizdavimas srityje:

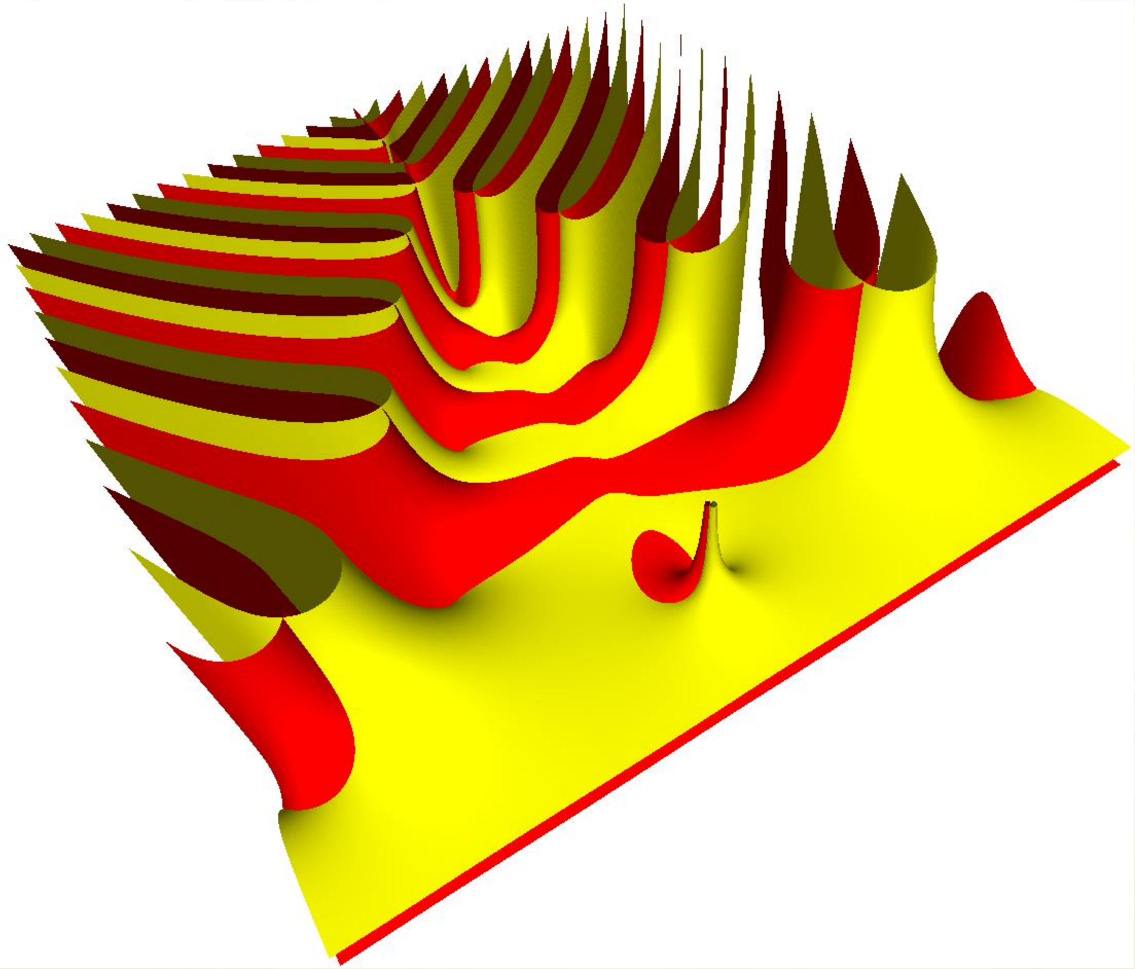
$$z_{min} \leq z \leq z_{max}$$

  - Efektyvus Rymano dzeta funkcijos atvaizdavimo algoritmas:
    - Konrado Knopo formulė (1930 m.) – globaliai konverguojanti eilutė,
    - Rymano-Zygelio formulė (1932 m.),
    - Zetafast algoritmas (2017 m.),
    - Kornelijaus Lancos algoritmas gama funkcijos apytikslei reikšmei apskaičiuoti (1964 m.).
  - Paviršių sankirtos generavimo algoritmas.

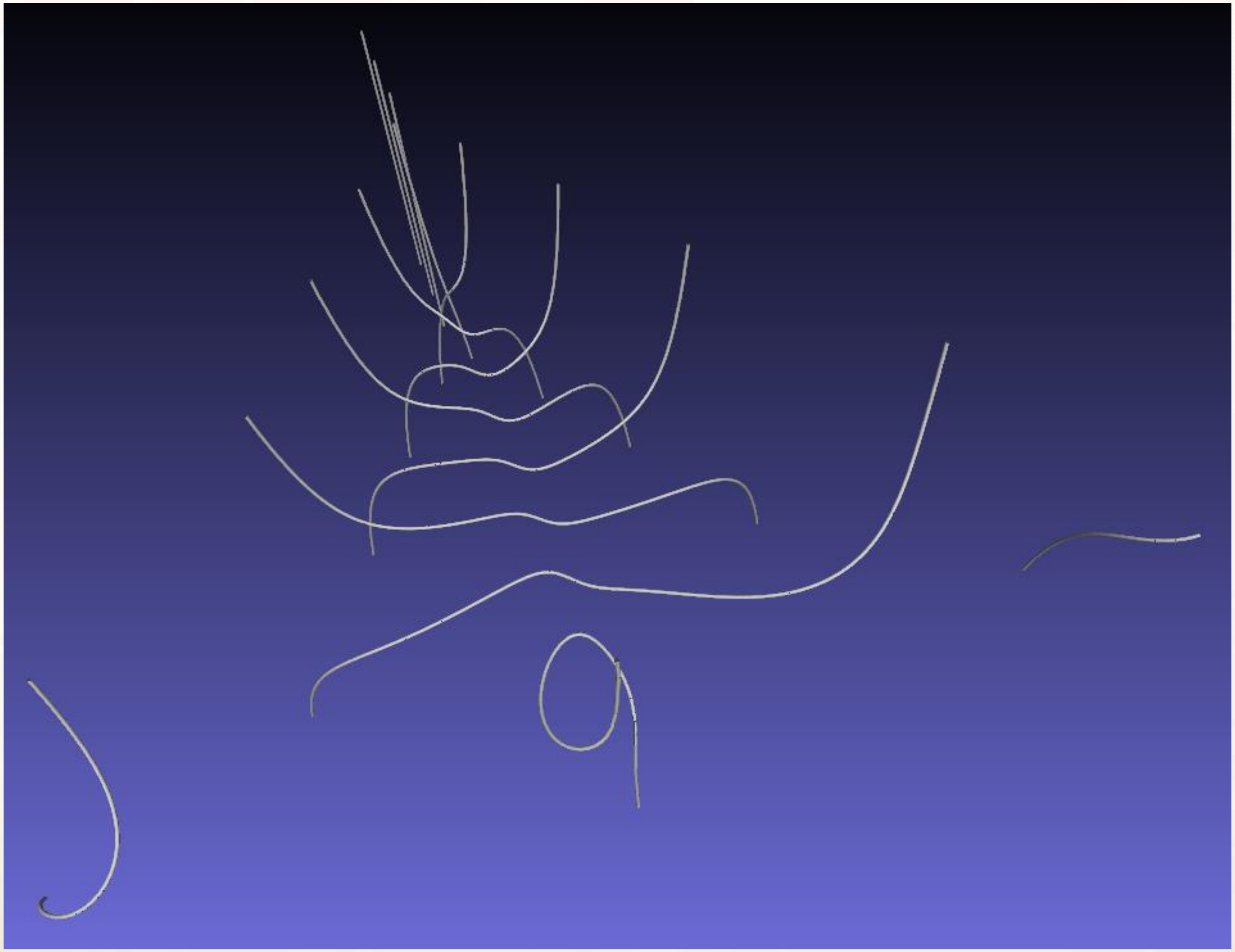
z = f(x,y) pjūviai



Dzeta funkcija



Paviršių sankirta



Millennium Prize Problems (2000)

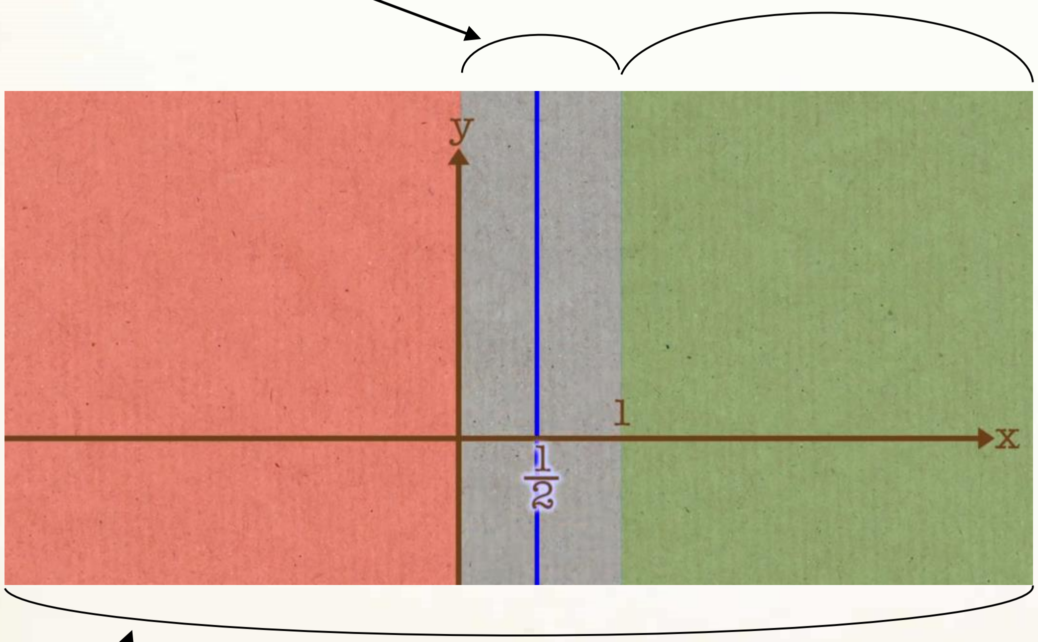
Clay Mathematics Institute, US.

- P versus NP problem
- Hodge conjecture
- Poincaré conjecture (solved in 2006)
- Riemann hypothesis
- Yang–Mills existence and mass gap
- Navier–Stokes existence and smoothness
- Birch and Swinnerton-Dyer conjecture

Rymano dzeta funkcija

$$\zeta(s) = \frac{1}{1 - 2^{1-s}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$$

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$



1826 m. – 1866 m.

$$\zeta(s) = 2^s \pi^{s-1} \ \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\ \zeta(1-s)$$

$$\Gamma(s) = \int_0^\infty x^{s-1}\, e^{-x}\, \mathrm{d}x$$

Ištrauka iš 1859 m. straipsnio

Betrachtet man nun das Integral

$$\int \frac{(-x)^{s-1}\,dx}{e^x - 1}$$

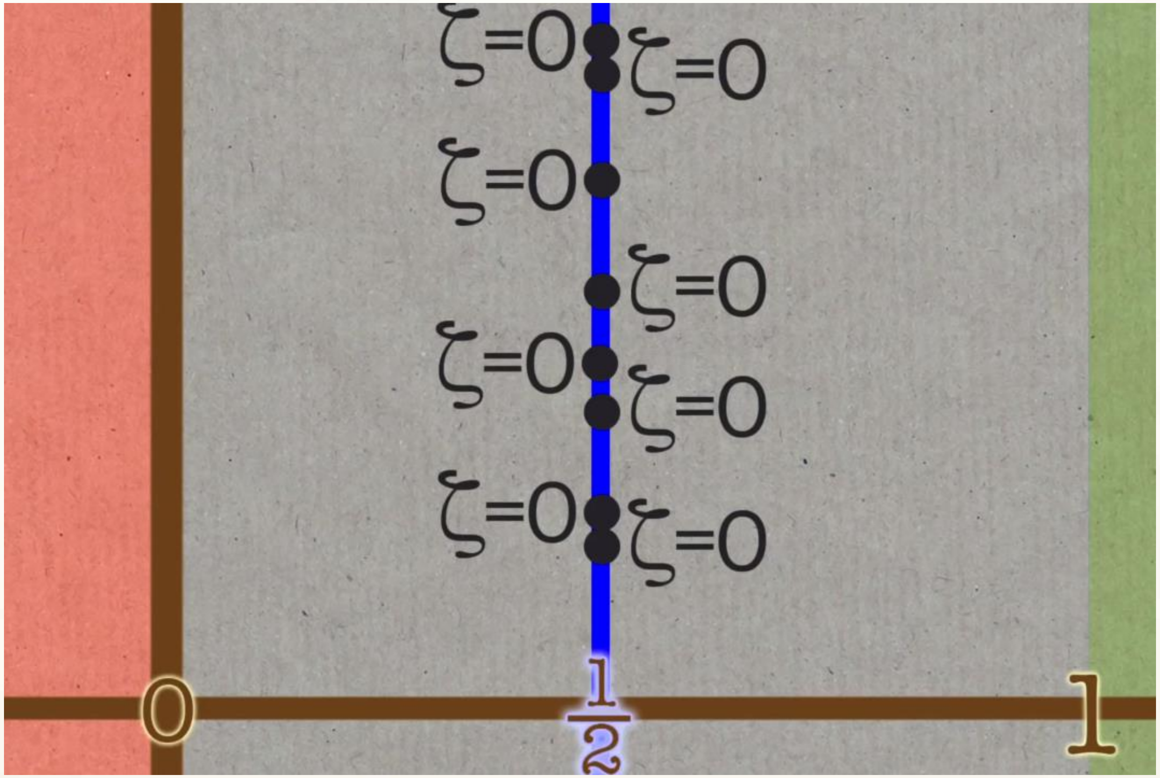
von $+\infty$ bis $+\infty$ positiv um ein Grössengebiet erstreckt, welches den Werth 0, aber keinen andern Unstetigkeitswerth der Function unter dem Integralzeichen im Innern enthält, so ergiebt sich dieses leicht gleich

$$\left(e^{-\pi s i} - e^{\pi s i}\right) \int_0^\infty \frac{x^{s-1}\,dx}{e^x - 1},$$

vorausgesetzt, dass in der vieldeutigen Function $(-x)^{s-1} = e^{(s-1)\log(-x)}$ der Logarithmus von $-x$ so bestimmt worden ist, dass er für ein negatives x reell wird. Man hat daher

$$2 \sin \pi s \; \Pi(s-1) \; \xi(s) = i \int_\infty^\infty \frac{(-x)^{s-1}\,dx}{e^x - 1},$$

Rymano hipotezė



Visi netrivialūs dzeta funkcijos nuliai yra tiesėje Re(ζ)=1/2.

Netrivialių nulių atradimo istorija

year	n	author
1903	15	J. P. Gram
1914	79	R. J. Backlund
1925	138	J. I. Hutchinson
1935	1041	E. C. Titchmarsh
1953	1104	A. M. Turing
1956	15 000	D. H. Lehmer
1956	25 000	D. H. Lehmer
1958	35 337	N. A. Meller
1966	250 000	R. S. Lehman
1968	3 500 000	J. B. Rosser, J. M. Yohe, L. Schoenfeld
1977	40 000 000	R. P. Brent
1979	81 000 001	R. P. Brent
1982	200 000 001	R. P. Brent, J. van de Lune, H. J. J. te Riele, D. T. Winter
1983	300 000 001	J. van de Lune, H. J. J. te Riele
1986	1 500 000 001	J. van de Lune, H. J. J. te Riele, D. T. Winter
2001	10 000 000 000	J. van de Lune (unpublished)
2004	900 000 000 000	S. Wedeniwski
2004	10 000 000 000 000	X. Gourdon and P. Demichel

Sąryšis su pirminiais skaičiais
(Oilerio daugybos formulė, 1737 m.)

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right) = \left(\prod_{p \text{ prime}} \frac{1}{1-p^{-s}}\right)^{-1} = \frac{1}{\zeta(s)}$$

Įrodymo idėja:

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \ldots$$

$$\frac{1}{2^s}\zeta(s) = \frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{6^s} + \frac{1}{8^s} + \frac{1}{10^s} + \ldots$$

Subtracting the second equation from the first we remove all elements that have a factor of 2:

$$\left(1 - \frac{1}{2^s}\right)\zeta(s) = 1 + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{9^s} + \frac{1}{11^s} + \frac{1}{13^s} + \ldots$$

Repeating for the next term:

$$\frac{1}{3^s}\left(1 - \frac{1}{2^s}\right)\zeta(s) = \frac{1}{3^s} + \frac{1}{9^s} + \frac{1}{15^s} + \frac{1}{21^s} + \frac{1}{27^s} + \frac{1}{33^s} + \ldots$$

Subtracting again we get:

$$\left(1 - \frac{1}{3^s}\right)\left(1 - \frac{1}{2^s}\right)\zeta(s) = 1 + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{11^s} + \frac{1}{13^s} + \frac{1}{17^s} + \ldots$$

Sąryšis su pirminiais skaičiais

Gramo eilutė:

$$R(x) = 1 + \sum_{k=1}^{\infty} \frac{(\ln x)^k}{k k! \zeta(k+1)}$$

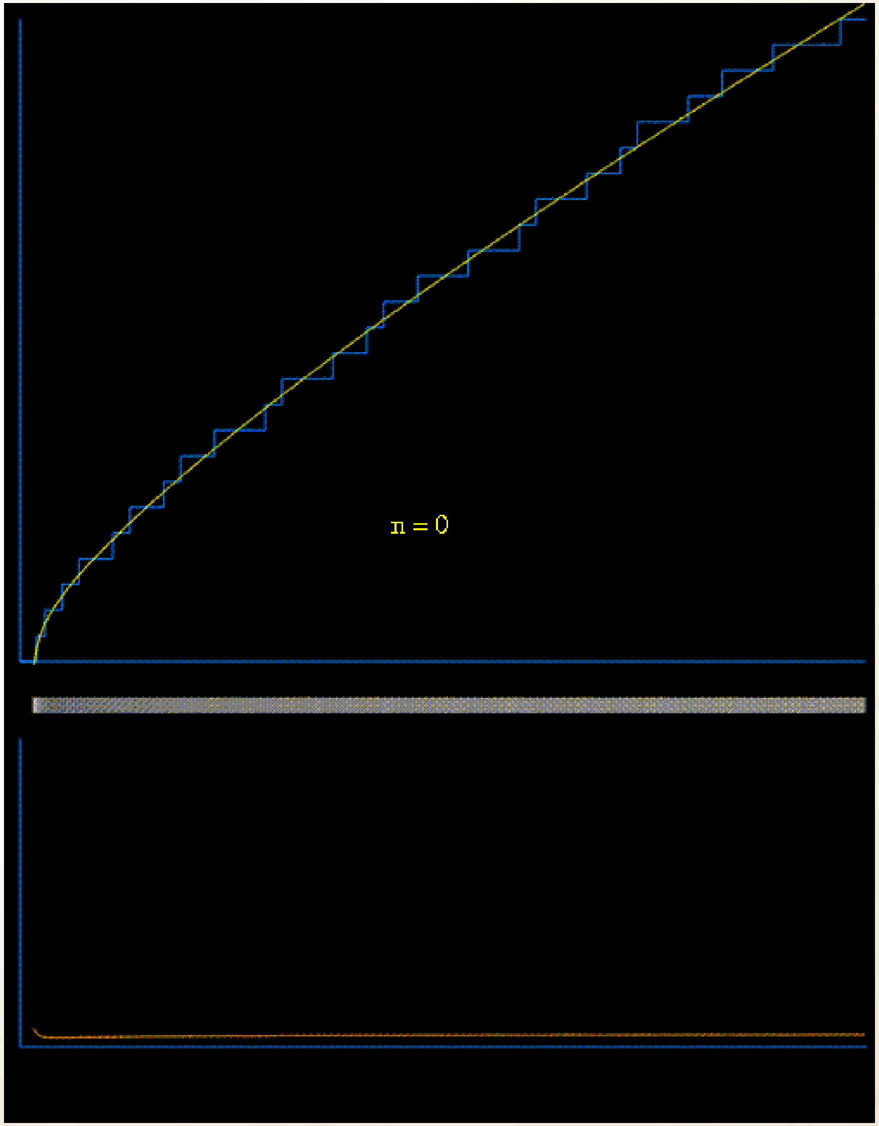
Sąryšis su pirminių skaičių pasiskirstymo funkcija:

$$\pi(x) = R(x) - \sum_{m=1}^{\infty} R(x^{-2m}) + \sum_{k=1}^{\infty} T_k(x)$$

$$R_n(x) = R(x) - \sum_{m=1}^{\infty} R(x^{-2m}) + \sum_{k=1}^{n} T_k(x)$$

$$T_k(x) = -R(x^{\rho_k}) - R(x^{\rho_{-k}})$$

ρ_k ir $\rho_{-k} = \overline{\rho_k}$ dzeta funkcijos netrivialių nulių poros (jungtiniai kompleksiniai sk.)



Globaliai konverguojančios eilutės

$$\zeta(s)=\frac{1}{s-1}\sum_{n=0}^{\infty}H_{n+1}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+2)^{1-s}$$

$$\zeta(s)=\frac{1}{s-1}\left\{-1+\sum_{n=0}^{\infty}H_{n+2}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+2)^{-s}\right\}$$

$$\zeta(s)=\frac{k!}{(s-k)_k}\sum_{n=0}^{\infty}\frac{1}{(n+k)!}\left[{n+k \atop n}\right]\sum_{\ell=0}^{n+k-1}(-1)^\ell\binom{n+k-1}{\ell}(\ell+1)^{k-s},\quad k=1,2,3,\ldots$$

$$\zeta(s)=\frac{1}{s-1}+\sum_{n=0}^{\infty}|G_{n+1}|\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+1)^{-s}$$

$$\zeta(s)=\frac{1}{s-1}+1-\sum_{n=0}^{\infty}C_{n+1}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+2)^{-s}$$

$$\zeta(s)=\frac{2(s-2)}{s-1}\zeta(s-1)+2\sum_{n=0}^{\infty}(-1)^nG_{n+2}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+1)^{-s}$$

$$\zeta(s)=-\sum_{l=1}^{k-1}\frac{(k-l+1)_l}{(s-l)_l}\zeta(s-l)+\frac{k}{s-k}+k\sum_{n=0}^{\infty}(-1)^nG_{n+1}^{(k)}\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+1)^{-s}$$

$$\zeta(s)=\frac{(a+1)^{1-s}}{s-1}+\sum_{n=0}^{\infty}(-1)^n\psi_{n+1}(a)\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+1)^{-s},\quad \Re(a)>-1$$

$$\zeta(s)=1+\frac{(a+2)^{1-s}}{s-1}+\sum_{n=0}^{\infty}(-1)^n\psi_{n+1}(a)\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+2)^{-s},\quad \Re(a)>-1$$

$$\zeta(s)=\frac{1}{a+\frac{1}{2}}\left\{-\frac{\zeta(s-1,1+a)}{s-1}+\zeta(s-1)+\sum_{n=0}^{\infty}(-1)^n\psi_{n+2}(a)\sum_{k=0}^{n}(-1)^k\binom{n}{k}(k+1)^{-s}\right\},\quad \Re(a)>-1$$

Konrado Knopo formulė

$$\zeta(s) = \frac{1}{1-2^{1-s}} \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{(k+1)^s}$$

...was conjectured by Konrad Knopp and proven by Helmut Hasse in 1930.

The series only appeared in an appendix to Hasse's paper, and did not become generally known until it was discussed by Jonathan Sondow in 1994.

Formulės taikymas dzeta funkcijos reikšmėms apskaičiuoti

$$\zeta(s) = \frac{1}{1-2^{1-s}} \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{(k+1)^s} =$$

$$= \frac{1}{1-2^{1-s}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)^s} \sum_{k=0}^{n} \binom{n}{k} \frac{1}{2^{k+1}} = \frac{1}{1-2^{1-s}} \lim_{k\to\infty} \sum_{n=1}^{k} \frac{1}{n^s}\, q_{n,k}$$

$$q_{n,k} = \frac{(-1)^{n+1}}{2^k}\, a_{n,k}$$

$$a_{k,k} = 1\ , \quad k \geq 1$$

$$a_{n,1} = 2\, a_{n-1,1} + 1\ , \quad n > 1$$

$$a_{n,k} = a_{n-1,\,k-1} + a_{n-1,\,k}\ , \quad 1 < n < k > 2$$

$a_{n,k}$ reikšmės

[1]
[3, 1]
[7, 4, 1]
[15, 11, 5, 1]
[31, 26, 16, 6, 1]
[63, 57, 42, 22, 7, 1]
[127, 120, 99, 64, 29, 8, 1]
[255, 247, 219, 163, 93, 37, 9, 1]
[511, 502, 466, 382, 256, 130, 46, 10, 1]
[1023, 1013, 968, 848, 638, 386, 176, 56, 11, 1]
[2047, 2036, 1981, 1816, 1486, 1024, 562, 232, 67, 12, 1]
[4095, 4083, 4017, 3797, 3302, 2510, 1586, 794, 299, 79, 13, 1]
[8191, 8178, 8100, 7814, 7099, 5812, 4096, 2380, 1093, 378, 92, 14, 1]
[16383, 16369, 16278, 15914, 14913, 12911, 9908, 6476, 3473, 1471, 470, 106, 15, 1]

$q_{n,k}$ reikšmės

$$\left[\frac{1}{2}\right]$$

$$\left[\frac{3}{4}, -\frac{1}{4}\right]$$

$$\left[\frac{7}{8}, -\frac{1}{2}, \frac{1}{8}\right]$$

$$\left[\frac{15}{16}, -\frac{11}{16}, \frac{5}{16}, -\frac{1}{16}\right]$$

$$\left[\frac{31}{32}, -\frac{13}{16}, \frac{1}{2}, -\frac{3}{16}, \frac{1}{32}\right]$$

$$\left[\frac{63}{64}, -\frac{57}{64}, \frac{21}{32}, -\frac{11}{32}, \frac{7}{64}, -\frac{1}{64}\right]$$

$$\left[\frac{127}{128}, -\frac{15}{16}, \frac{99}{128}, -\frac{1}{2}, \frac{29}{128}, -\frac{1}{16}, \frac{1}{128}\right]$$

$$\left[\frac{255}{256}, -\frac{247}{256}, \frac{219}{256}, -\frac{163}{256}, \frac{93}{256}, -\frac{37}{256}, \frac{9}{256}, -\frac{1}{256}\right]$$

$$\left[\frac{511}{512}, -\frac{251}{256}, \frac{233}{256}, -\frac{191}{256}, \frac{1}{2}, -\frac{65}{256}, \frac{23}{256}, -\frac{5}{256}, \frac{1}{512}\right]$$

$$\left[\frac{1023}{1024}, -\frac{1013}{1024}, \frac{121}{128}, -\frac{53}{64}, \frac{319}{512}, -\frac{193}{512}, \frac{11}{64}, -\frac{7}{128}, \frac{11}{1024}, -\frac{1}{1024}\right]$$

$$\left[\frac{2047}{2048}, -\frac{509}{512}, \frac{1981}{2048}, -\frac{227}{256}, \frac{743}{1024}, -\frac{1}{2}, \frac{281}{1024}, -\frac{29}{256}, \frac{67}{2048}, -\frac{3}{512}, \frac{1}{2048}\right]$$

$$\left[\frac{4095}{4096}, -\frac{4083}{4096}, \frac{4017}{4096}, -\frac{3797}{4096}, \frac{1651}{2048}, -\frac{1255}{2048}, \frac{793}{2048}, -\frac{397}{2048}, \frac{299}{4096}, -\frac{79}{4096}, \frac{13}{4096}, -\frac{1}{4096}\right]$$

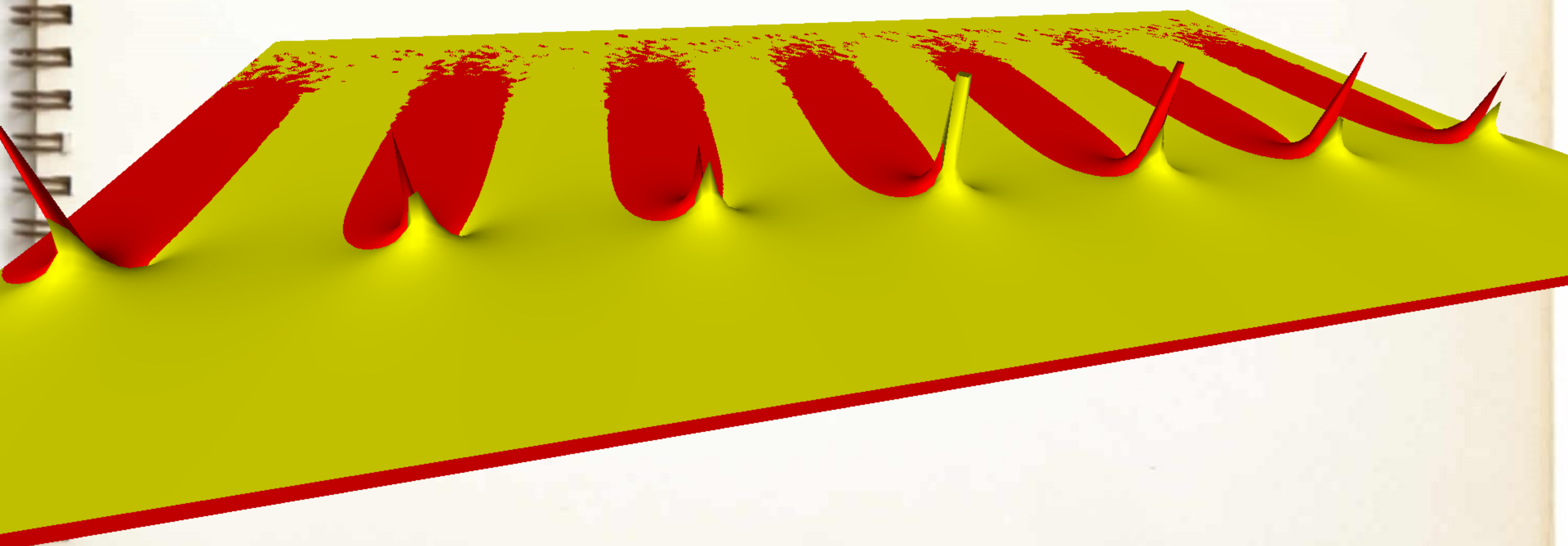
$$\left[\frac{8191}{8192}, -\frac{4089}{4096}, \frac{2025}{2048}, -\frac{3907}{4096}, \frac{7099}{8192}, -\frac{1453}{2048}, \frac{1}{2}, -\frac{595}{2048}, \frac{1093}{8192}, -\frac{189}{4096}, \frac{23}{2048}, -\frac{7}{4096}, \frac{1}{8192}\right]$$

$$\left[\frac{16383}{16384}, -\frac{16369}{16384}, \frac{8139}{8192}, -\frac{7957}{8192}, \frac{14913}{16384}, -\frac{12911}{16384}, \frac{2477}{4096}, -\frac{1619}{4096}, \frac{3473}{16384}, -\frac{1471}{16384}, \frac{235}{8192}, -\frac{53}{8192}, \frac{15}{16384}, -\frac{1}{16384}\right]$$

Eilutės konvergavimas į dzeta funkciją

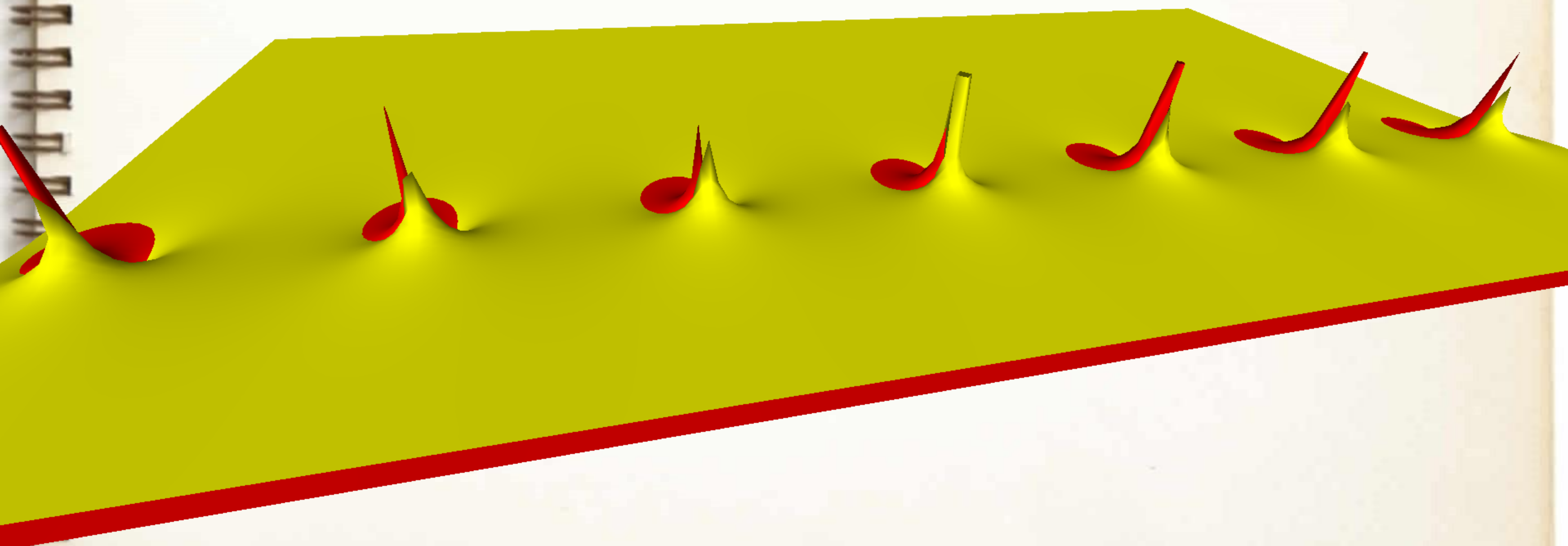
k = 1

Eilutės konvergavimas į dzeta funkciją



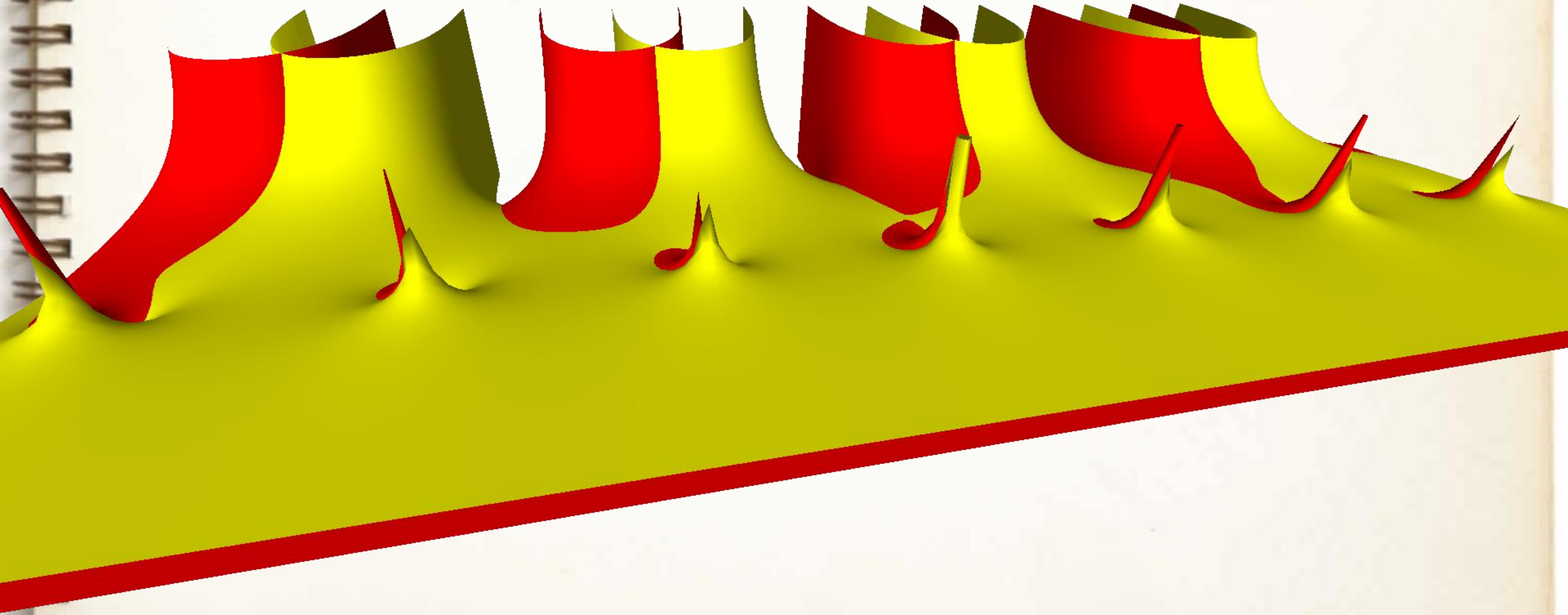
k = 2

Eilutės konvergavimas į dzeta funkciją



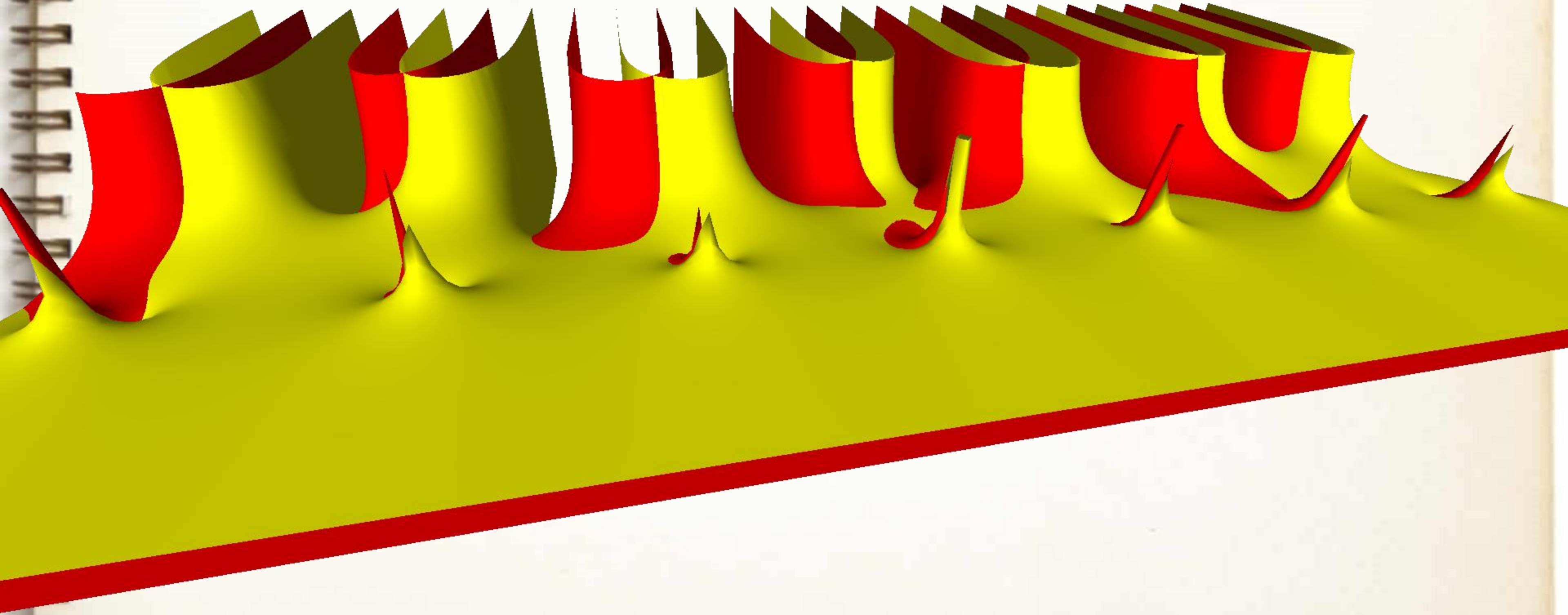
k = 3

Eilutės konvergavimas į dzeta funkciją



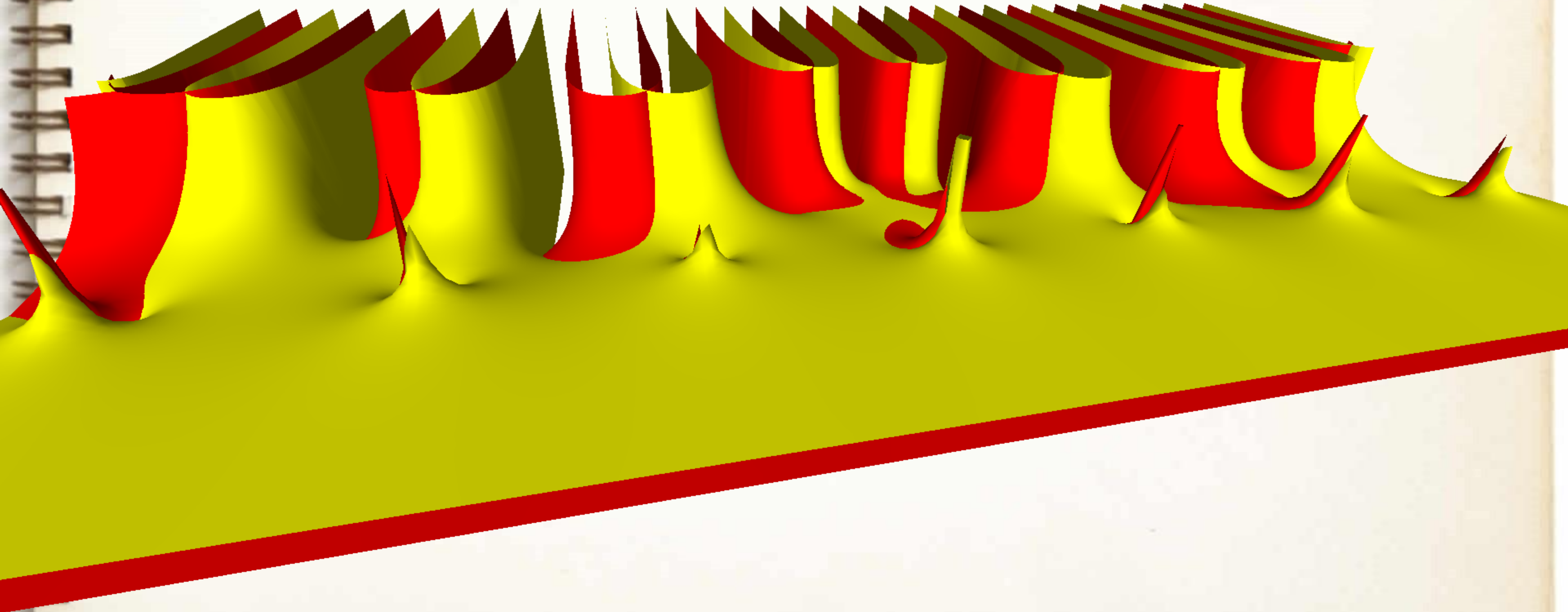
k = 4

Eilutės konvergavimas į dzeta funkciją



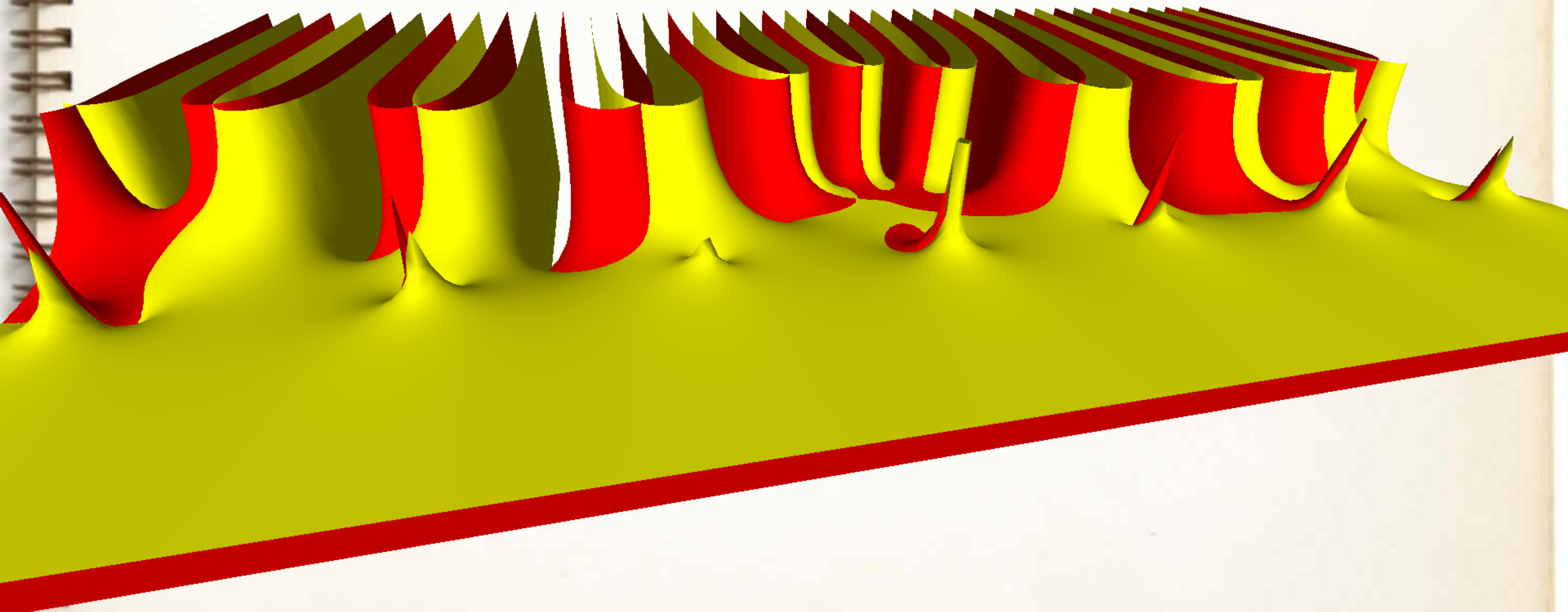
k = 5

Eilutės konvergavimas į dzeta funkciją



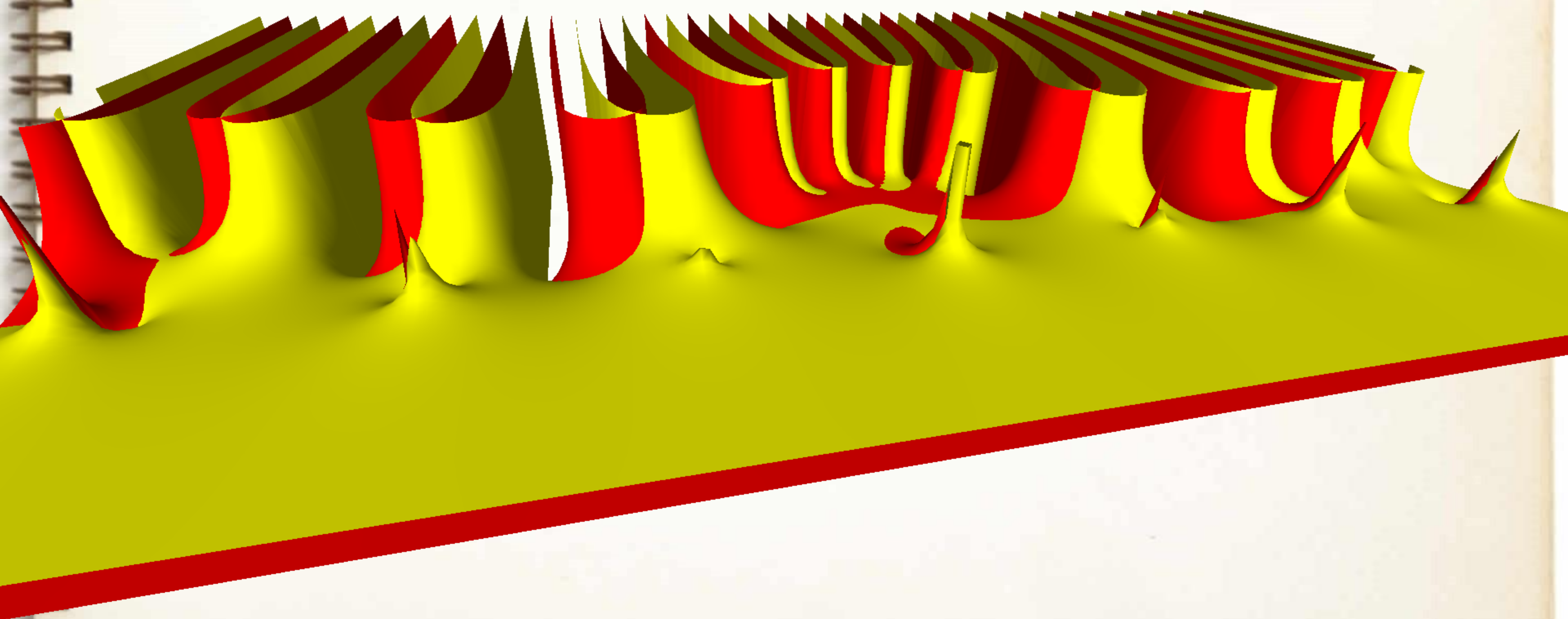
k = 6

Eilutės konvergavimas į dzeta funkciją



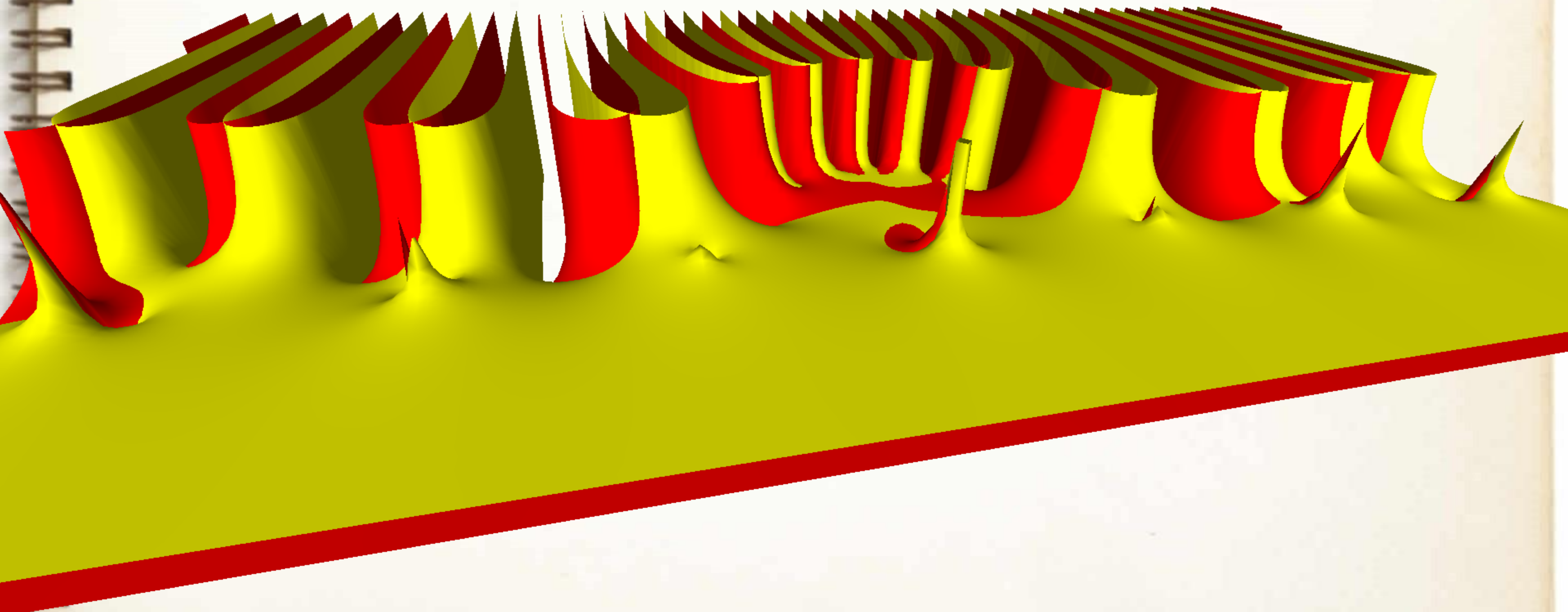
k = 7

Eilutės konvergavimas į dzeta funkciją



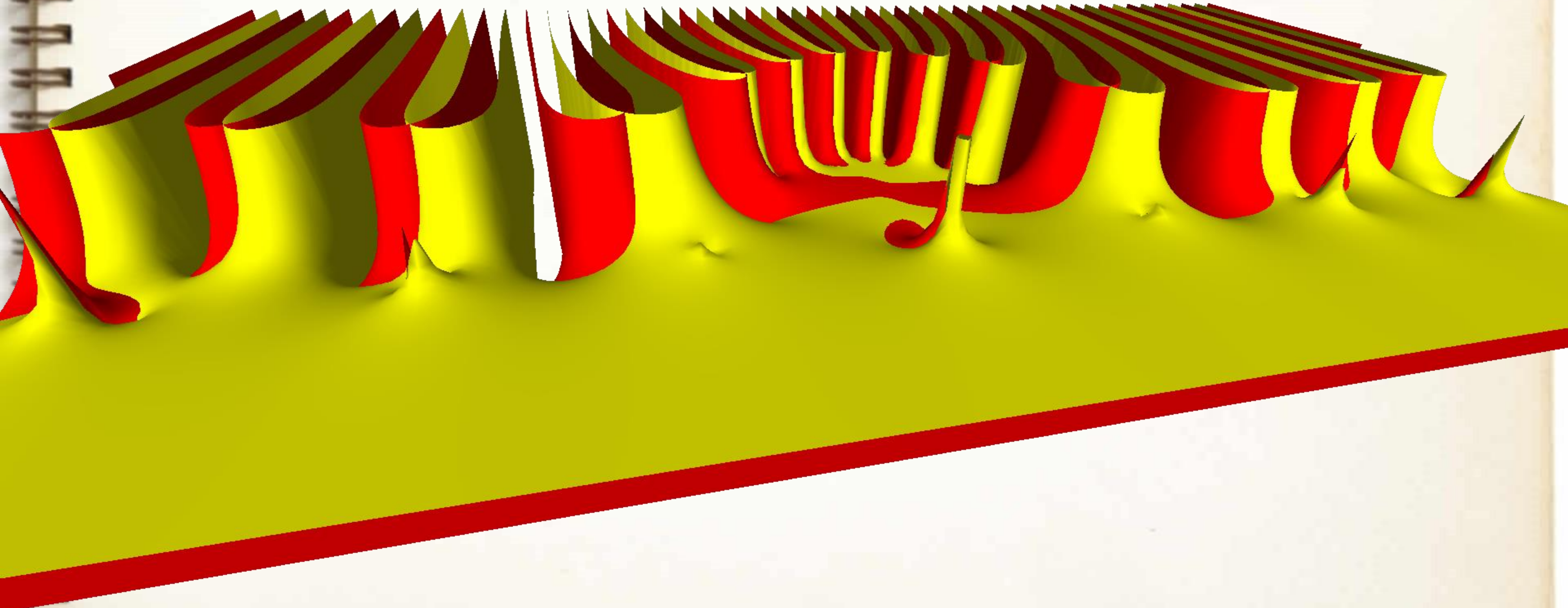
k = 8

Eilutės konvergavimas į dzeta funkciją



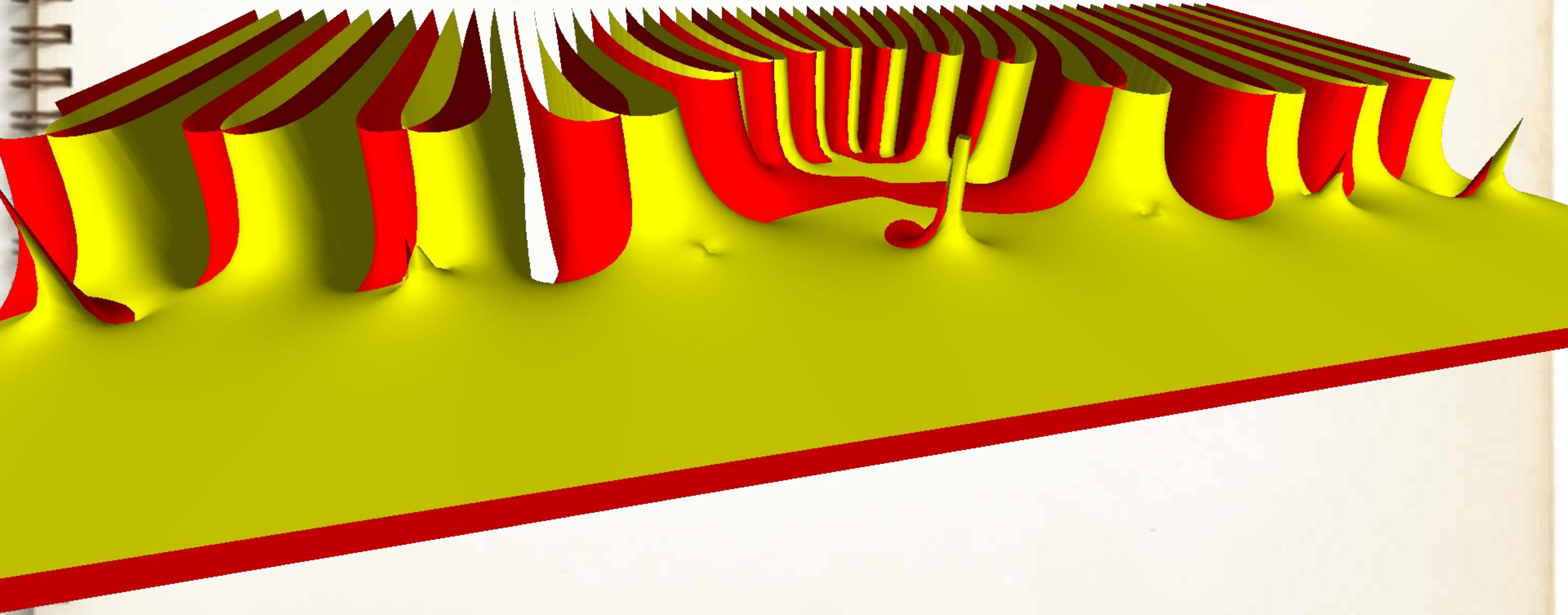
k = 9

Eilutės konvergavimas į dzeta funkciją



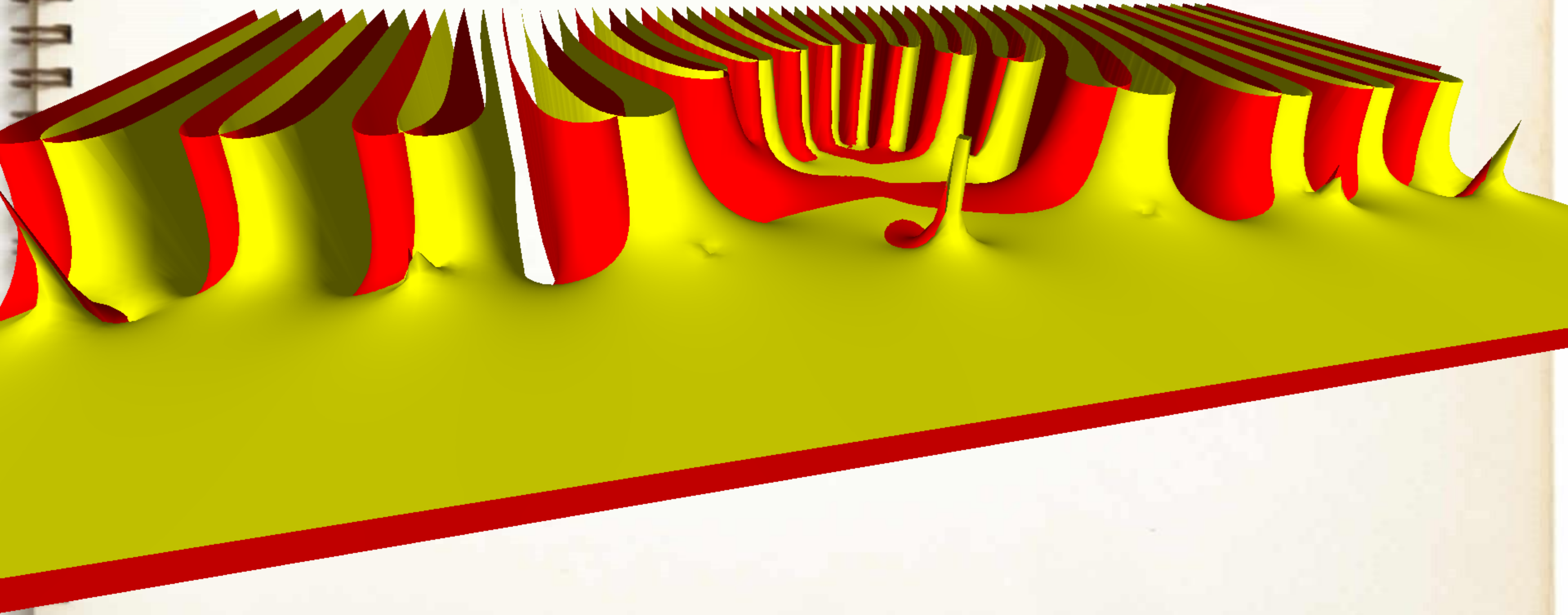
k = 10

Eilutės konvergavimas į dzeta funkciją



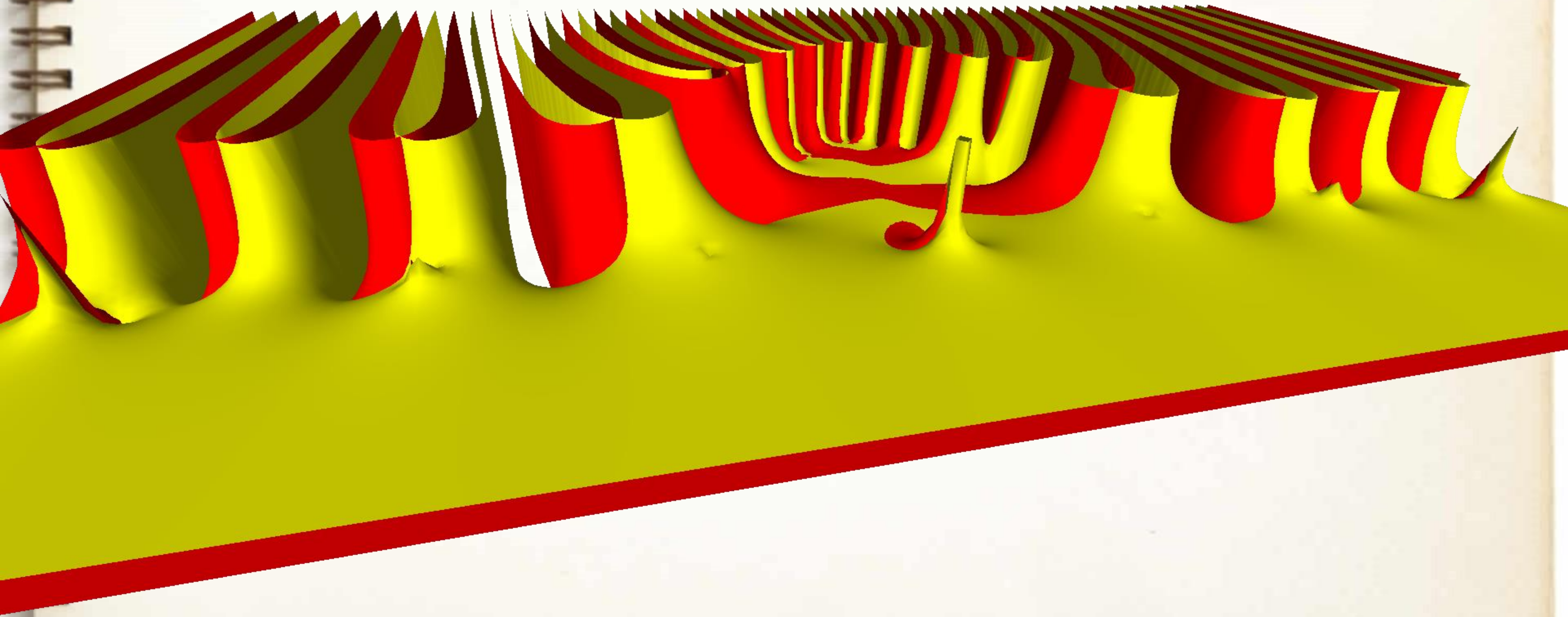
k = 11

Eilutės konvergavimas į dzeta funkciją



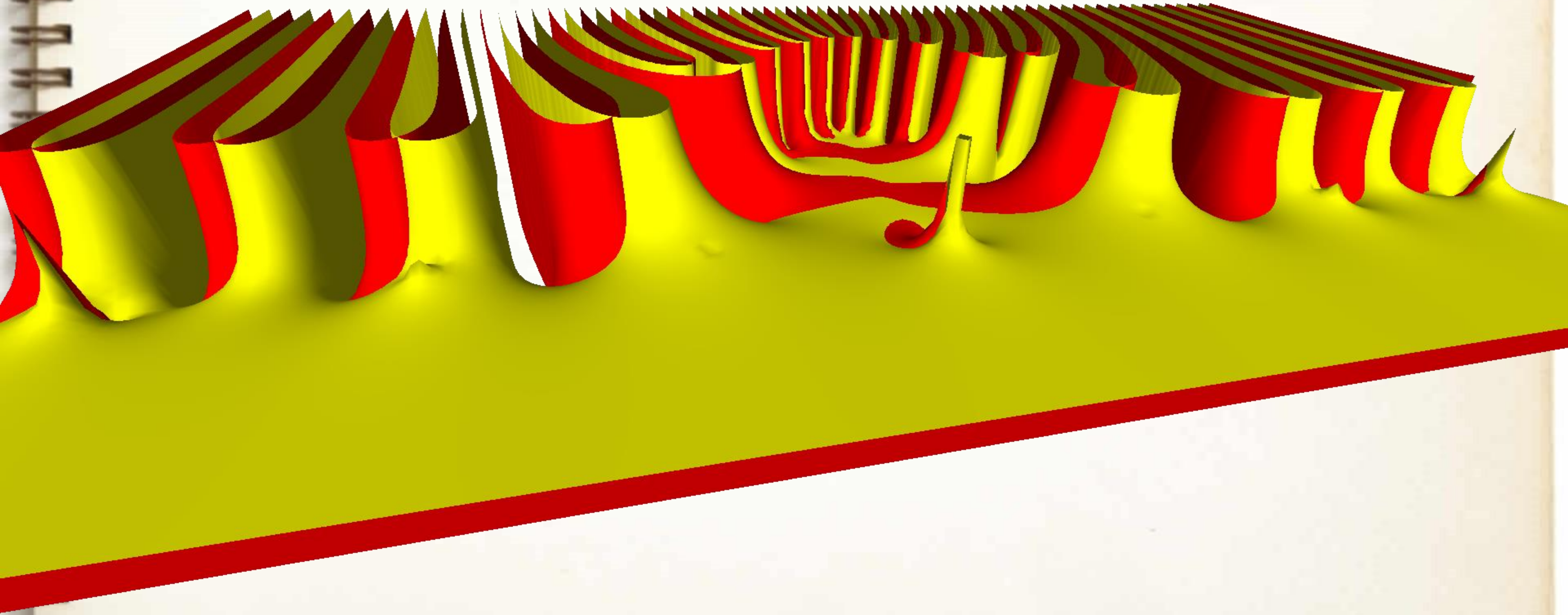
k = 12

Eilutės konvergavimas į dzeta funkciją



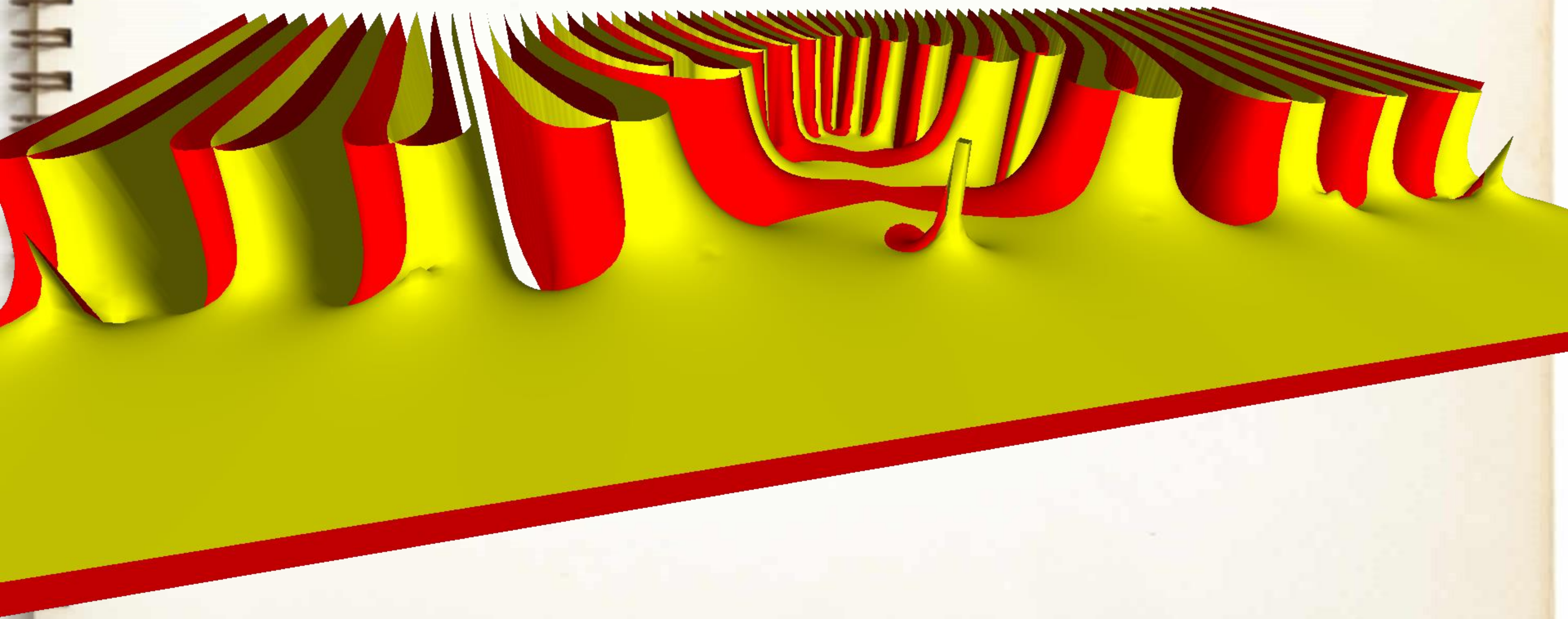
k = 13

Eilutės konvergavimas į dzeta funkciją



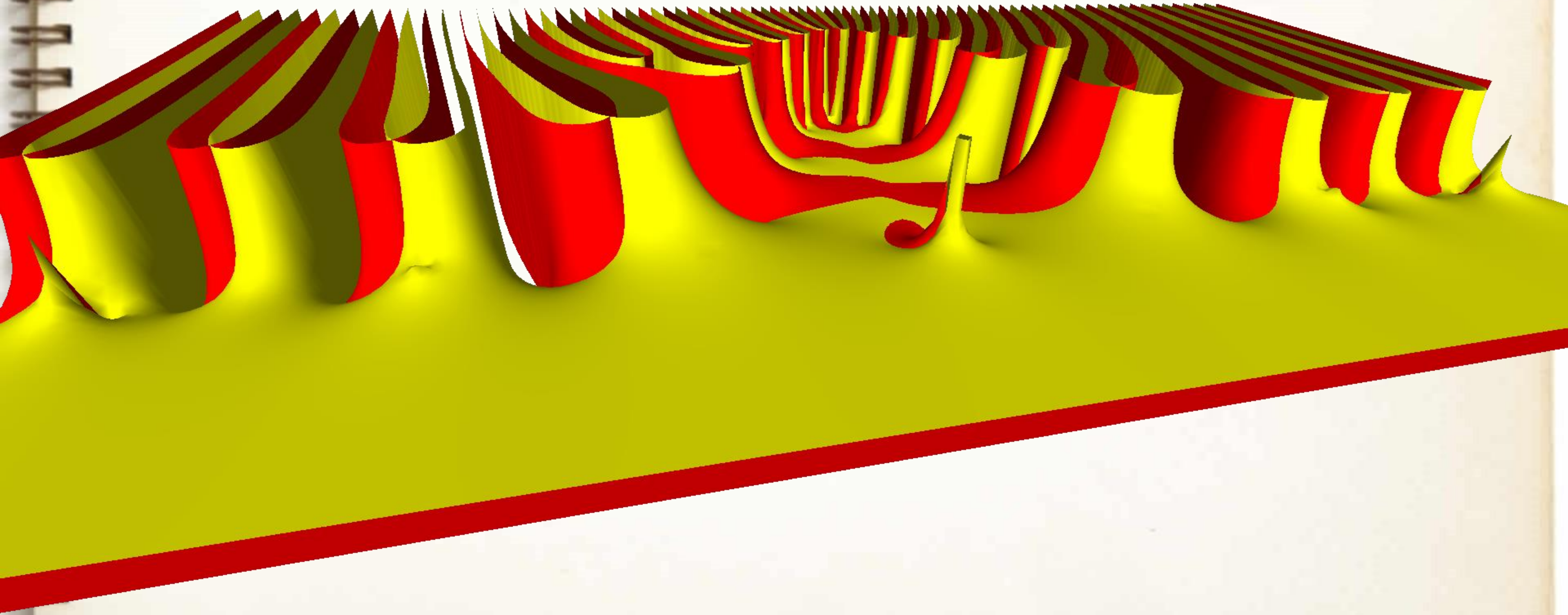
k = 14

Eilutės konvergavimas į dzeta funkciją



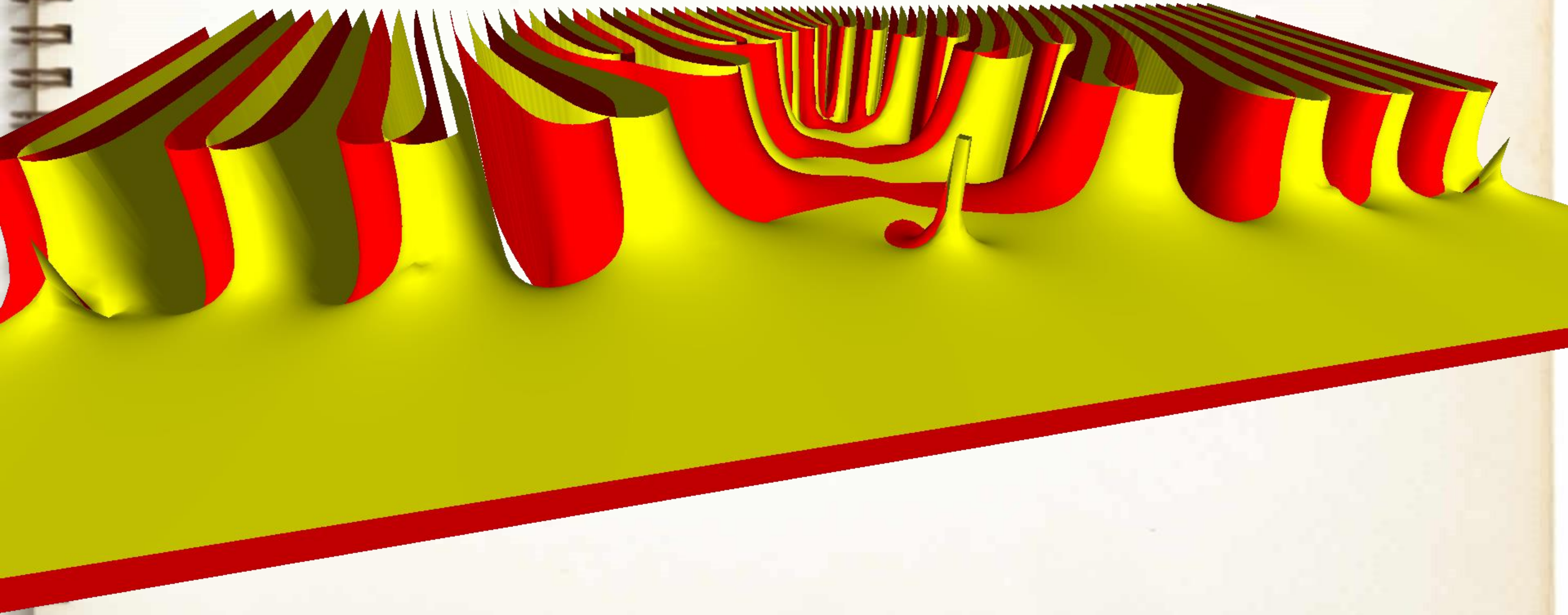
k = 15

Eilutės konvergavimas į dzeta funkciją



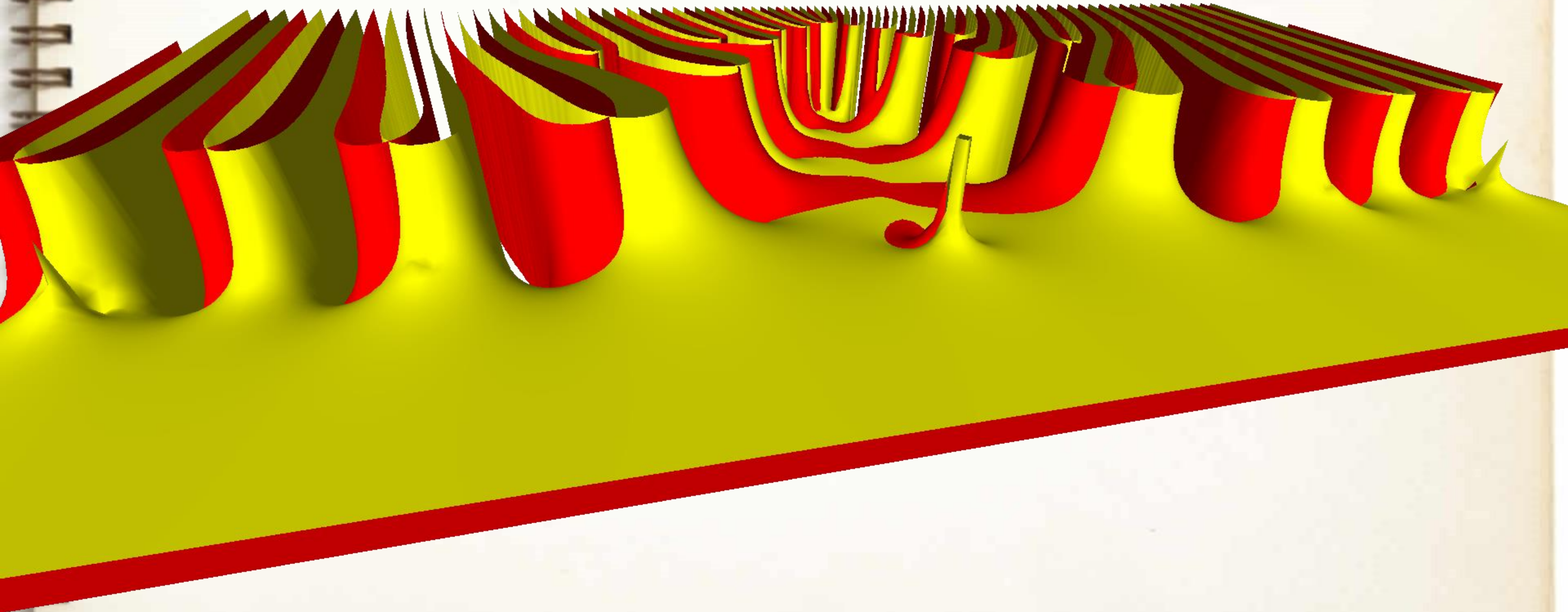
k = 16

Eilutės konvergavimas į dzeta funkciją



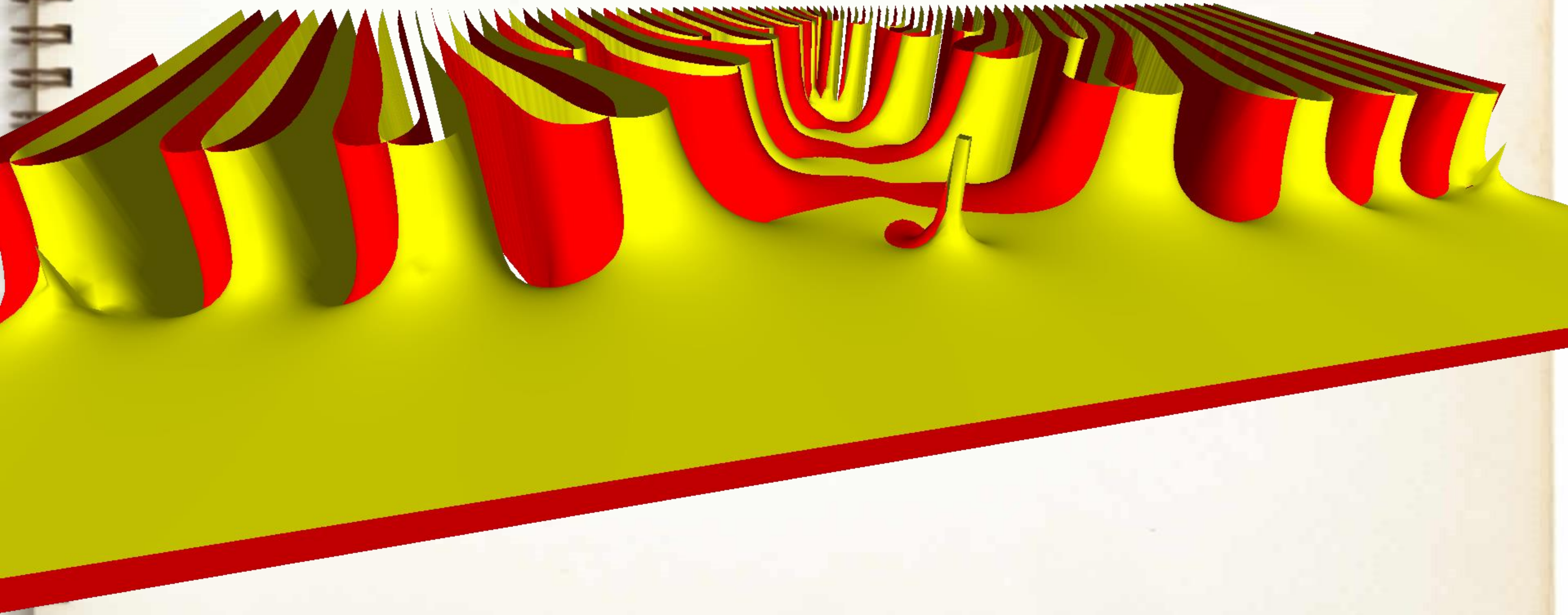
k = 17

Eilutės konvergavimas į dzeta funkciją



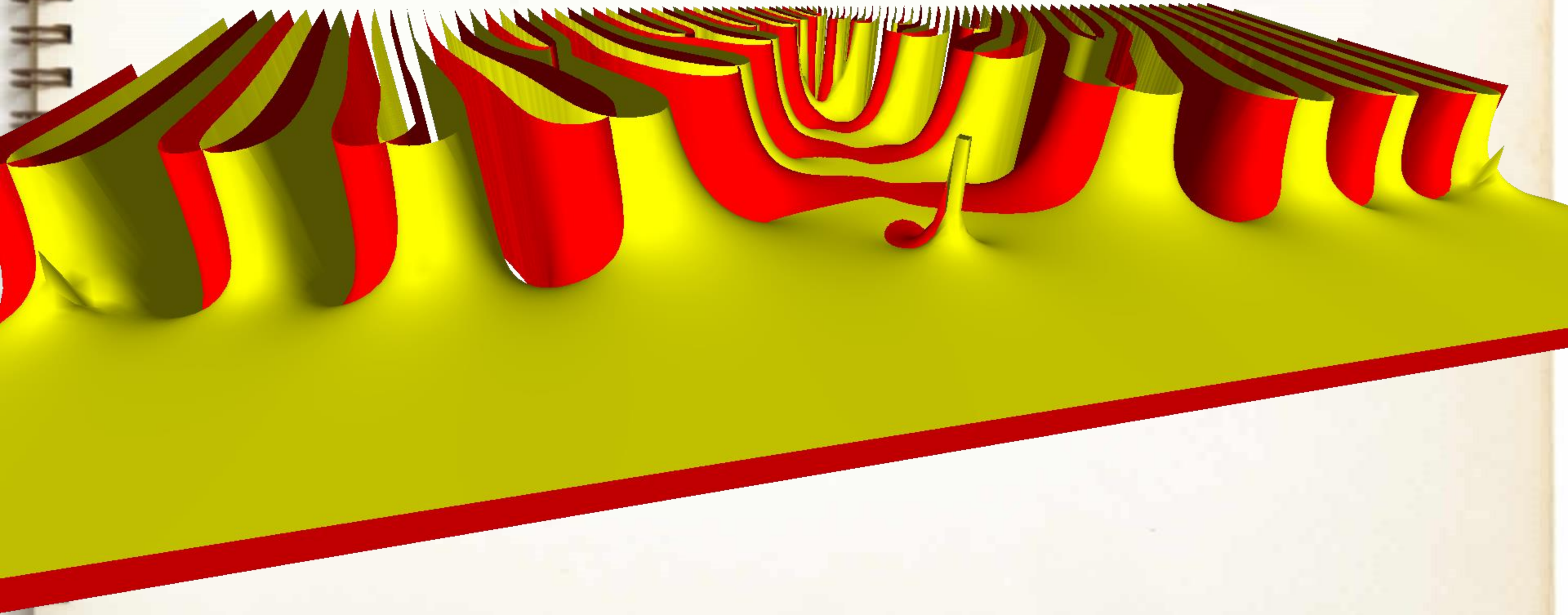
k = 18

Eilutės konvergavimas į dzeta funkciją



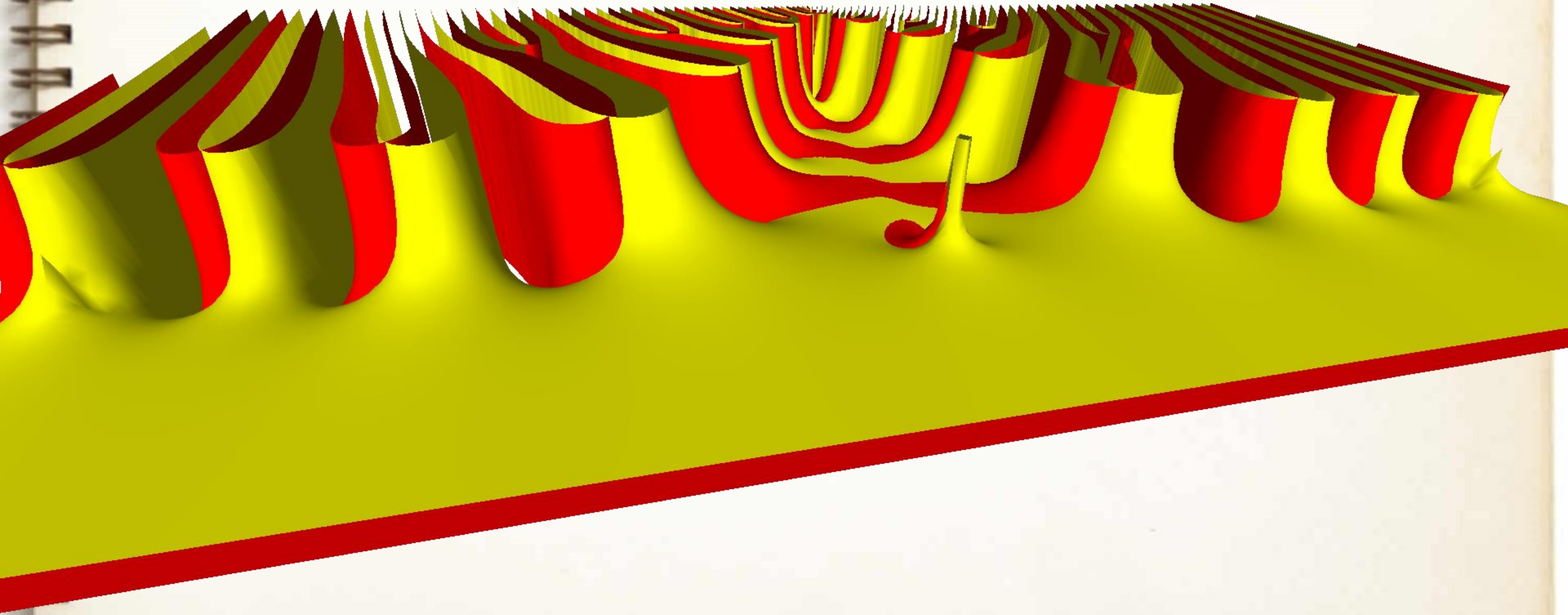
k = 19

Eilutės konvergavimas į dzeta funkciją



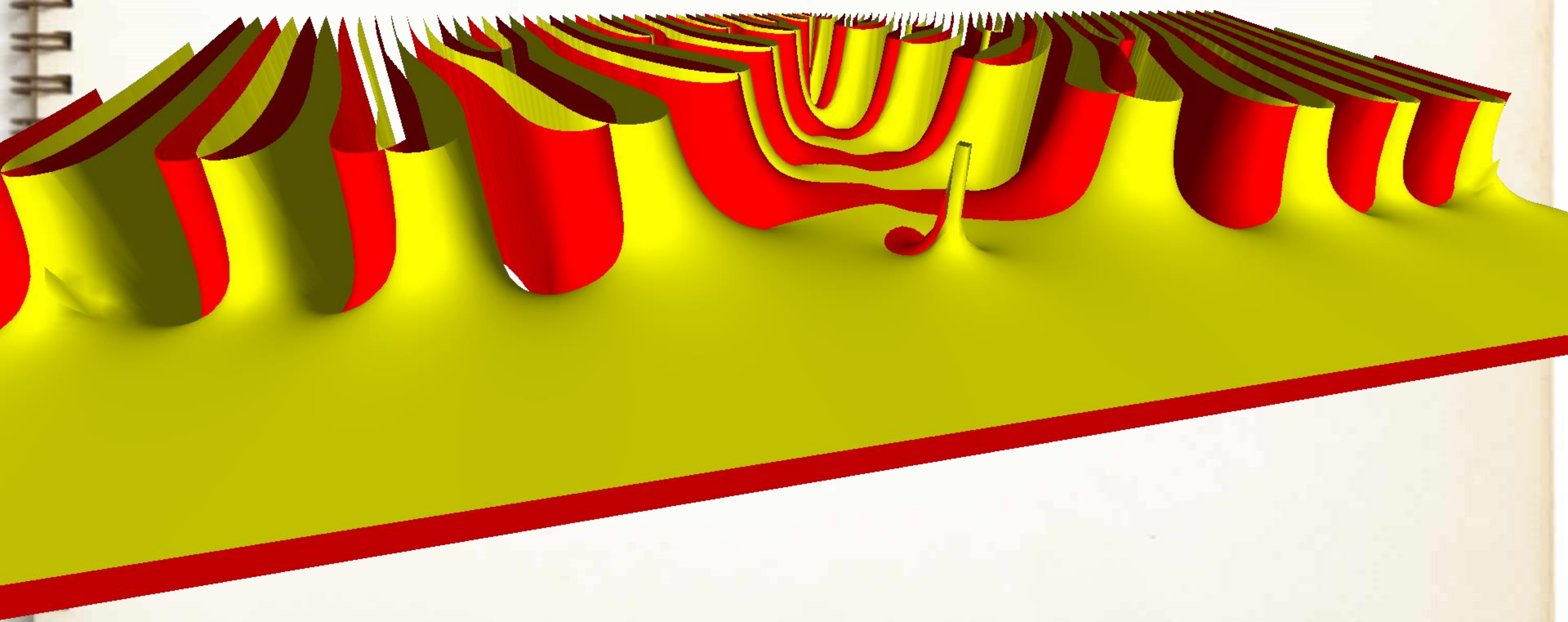
k = 20

Eilutės konvergavimas į dzeta funkciją



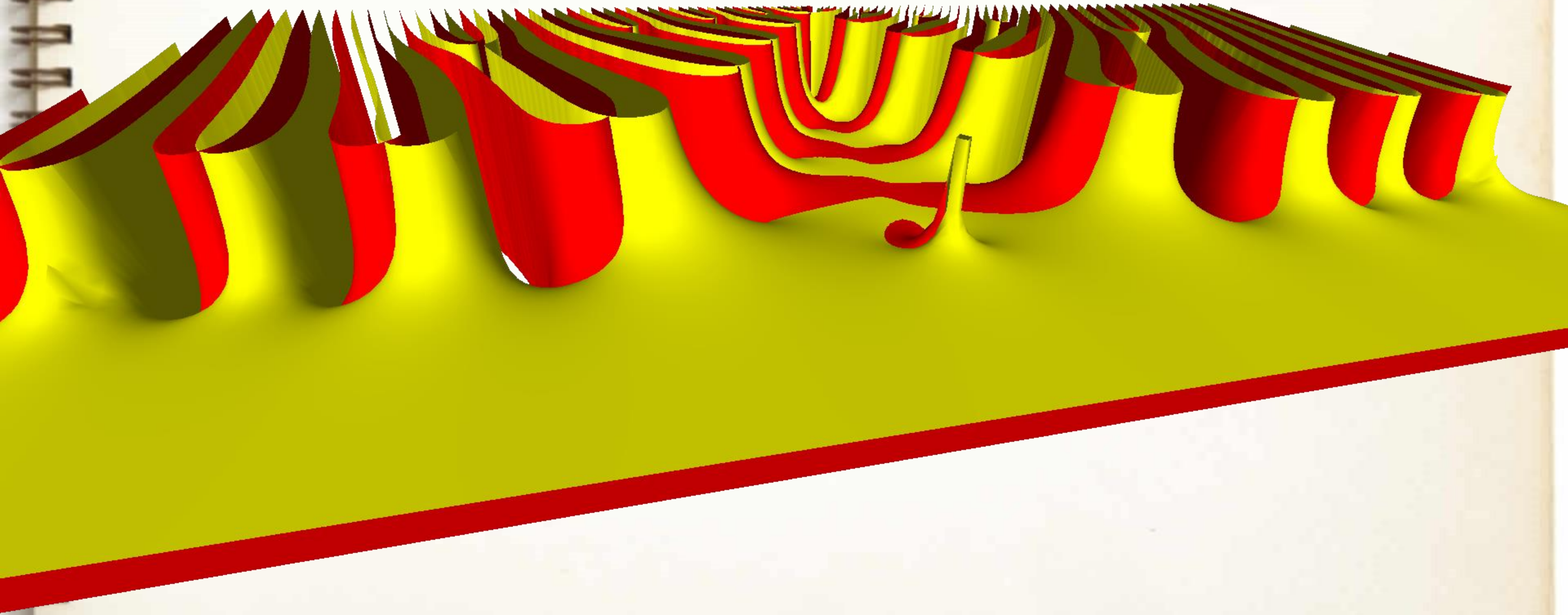
k = 21

Eilutės konvergavimas į dzeta funkciją



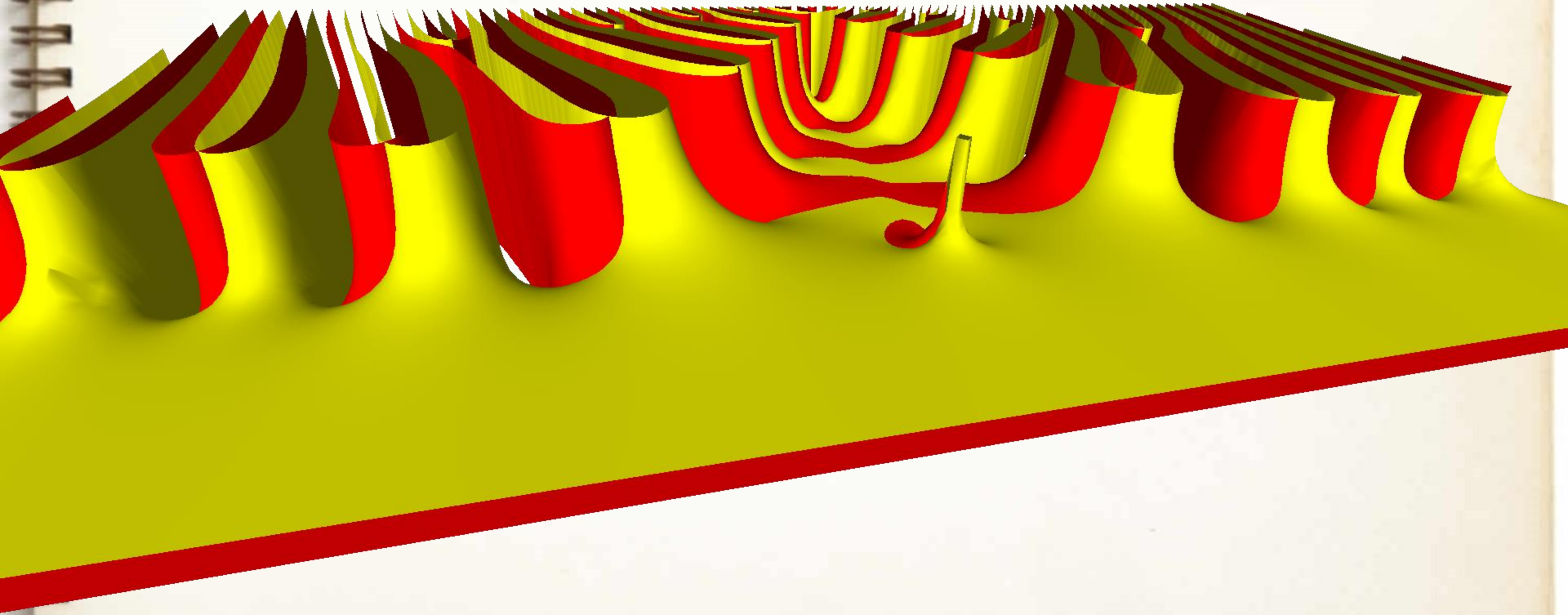
k = 22

Eilutės konvergavimas į dzeta funkciją



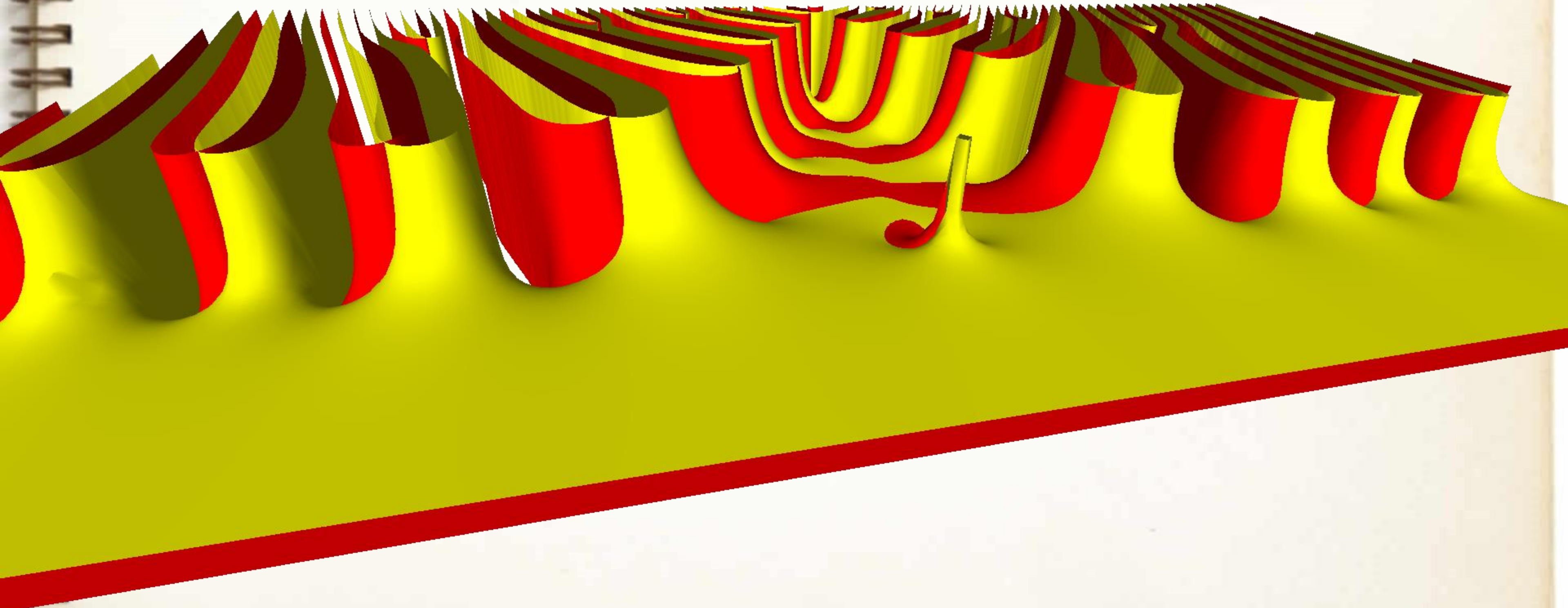
k = 23

Eilutės konvergavimas į dzeta funkciją



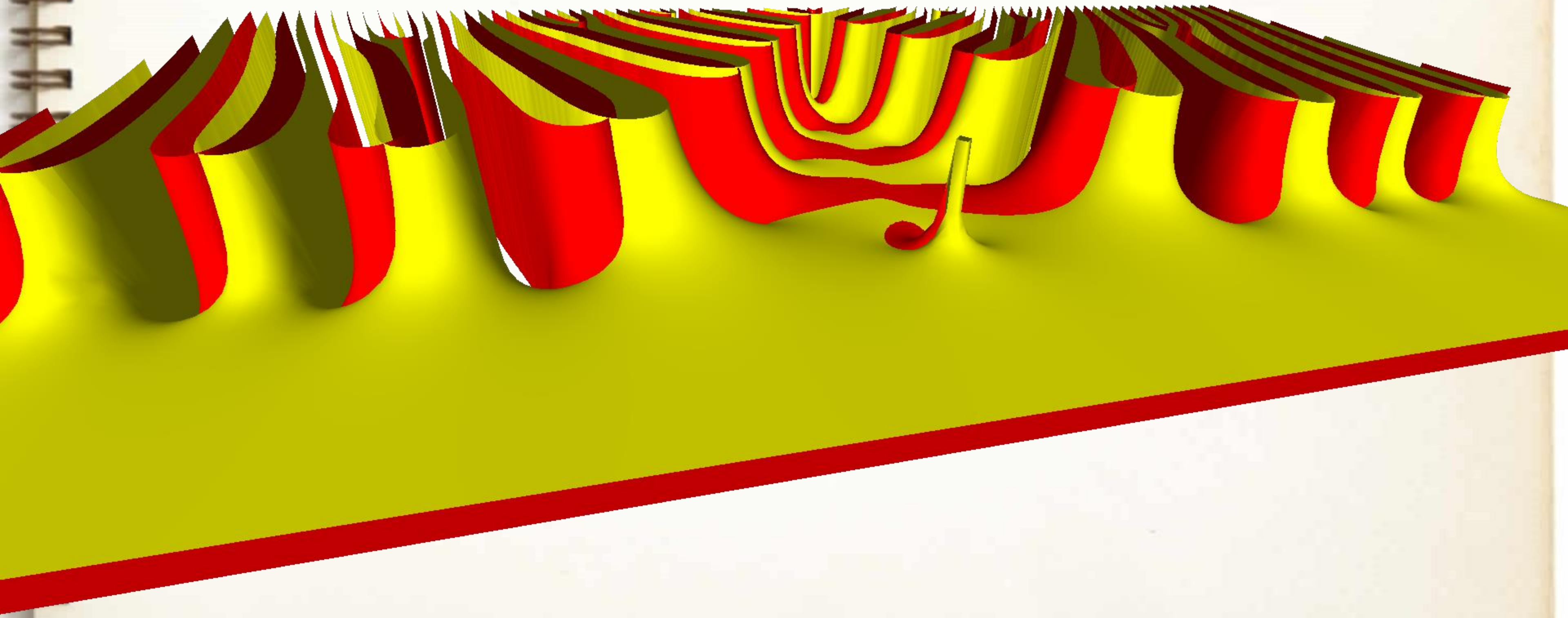
k = 24

Eilutės konvergavimas į dzeta funkciją



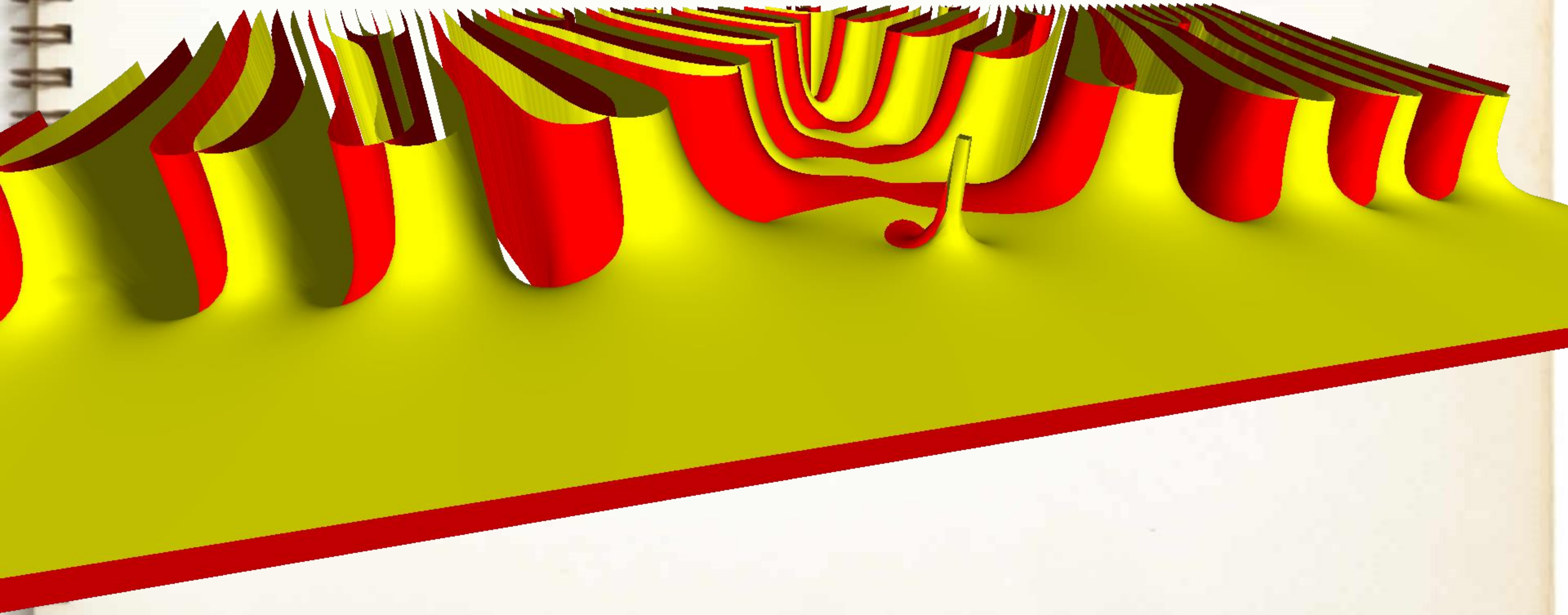
k = 25

Eilutės konvergavimas į dzeta funkciją



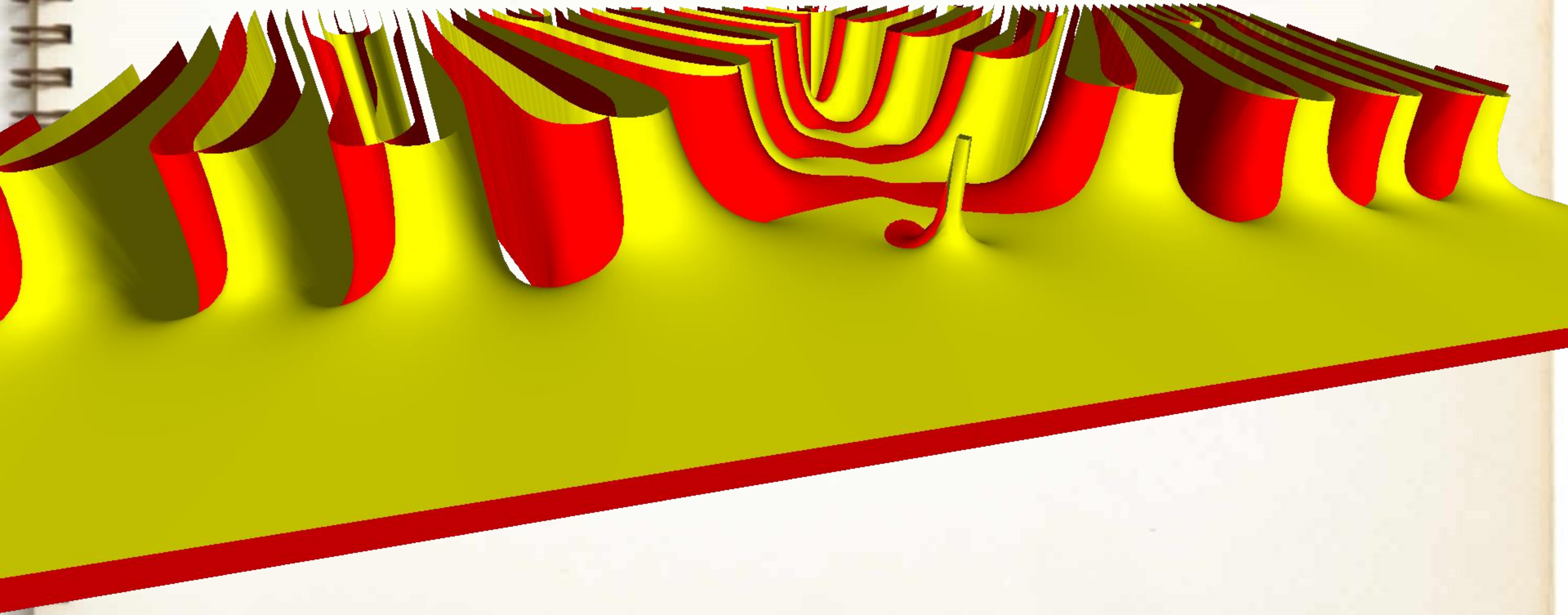
k = 26

Eilutės konvergavimas į dzeta funkciją



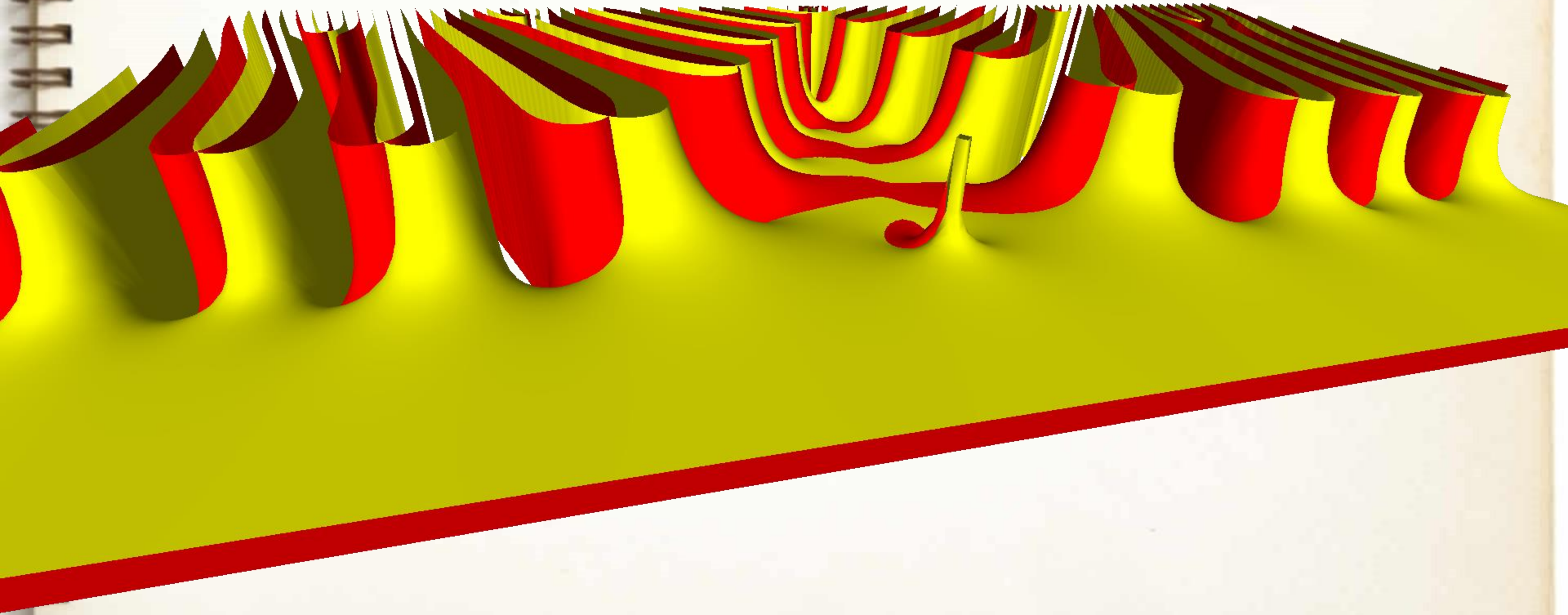
k = 27

Eilutės konvergavimas į dzeta funkciją



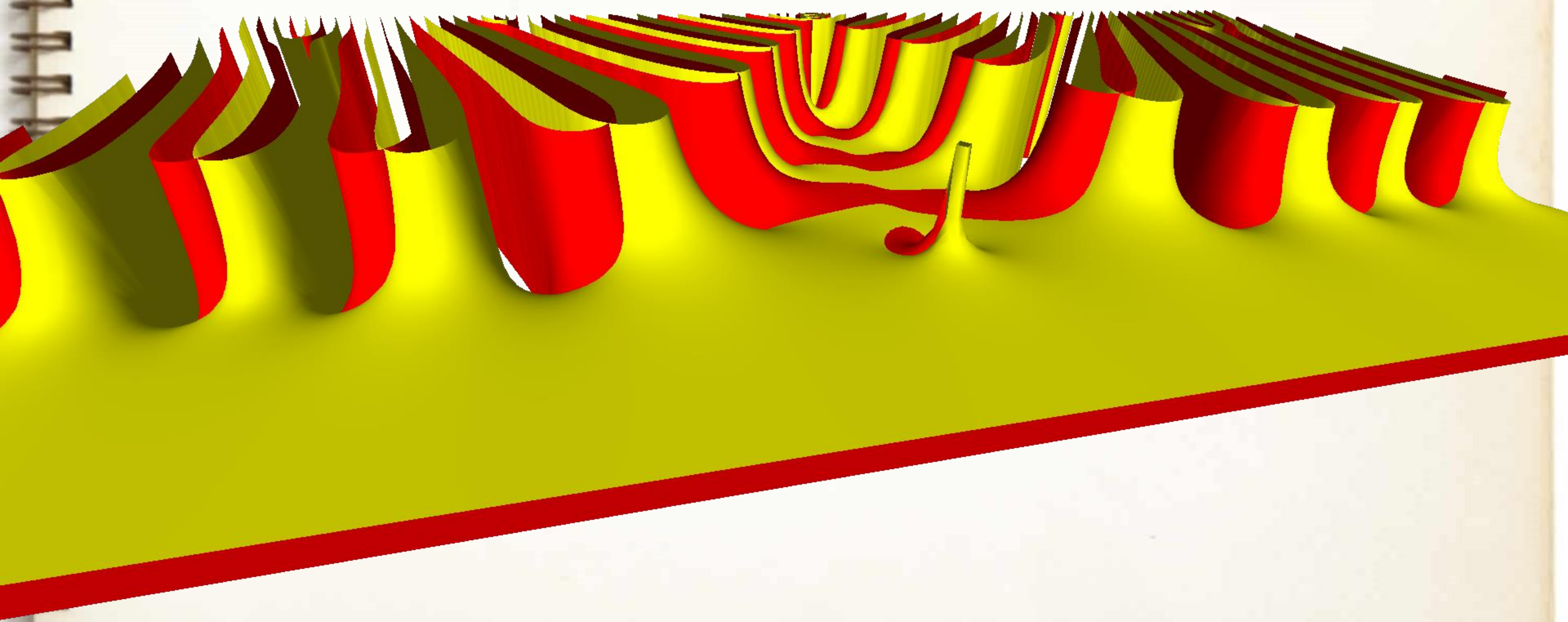
k = 28

Eilutės konvergavimas į dzeta funkciją



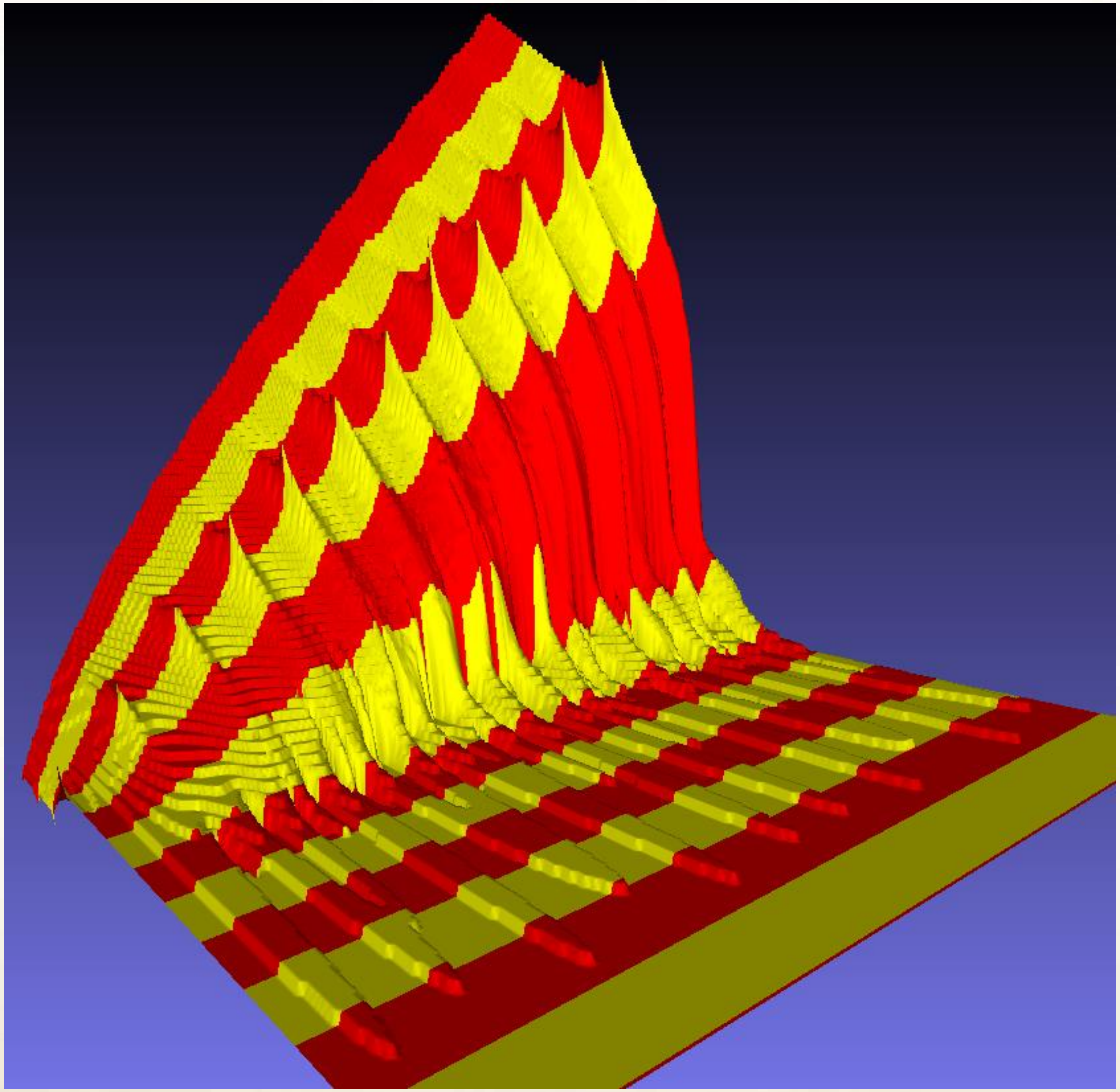
k = 29

Eilutės konvergavimas į dzeta funkciją

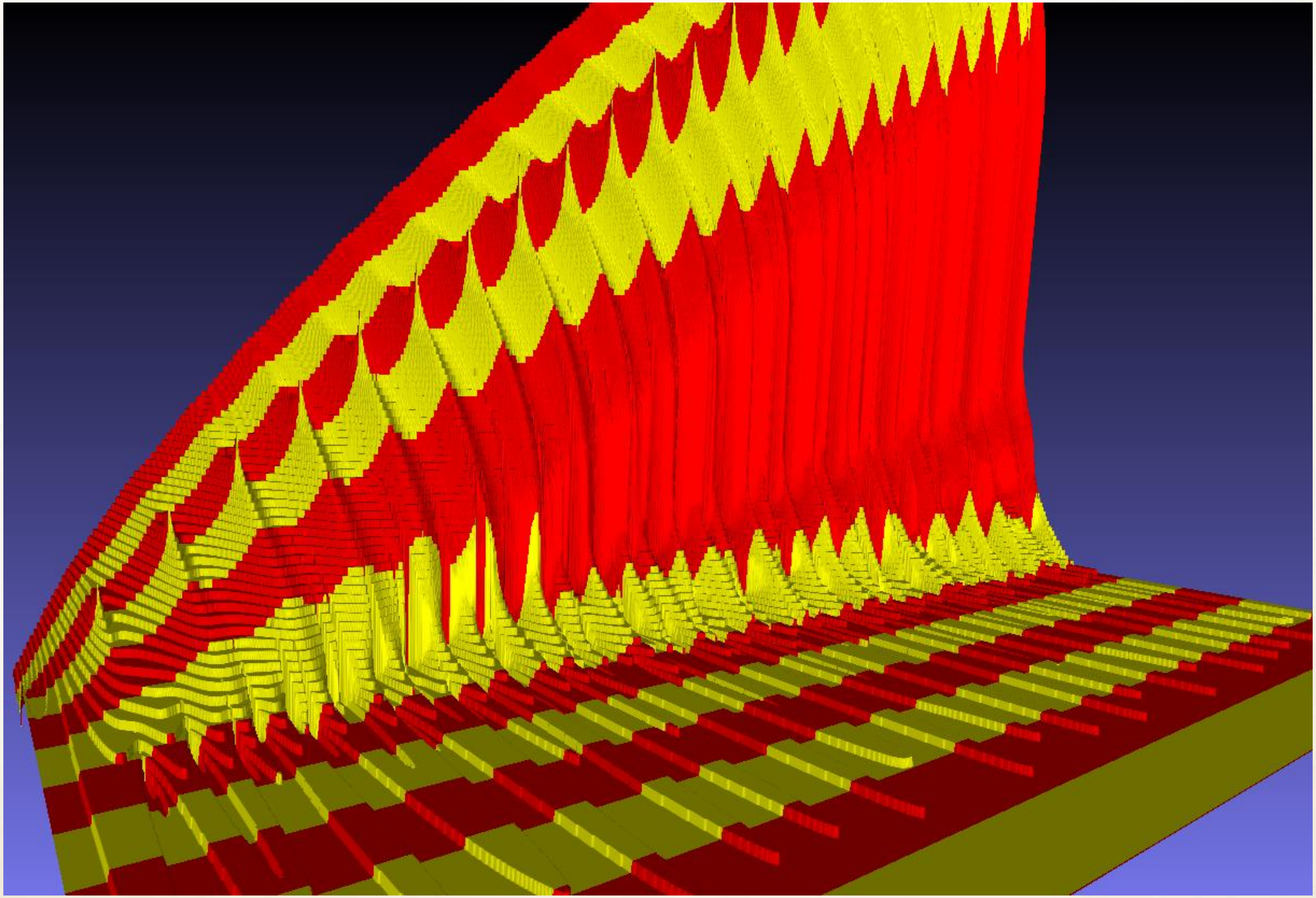


k = 30

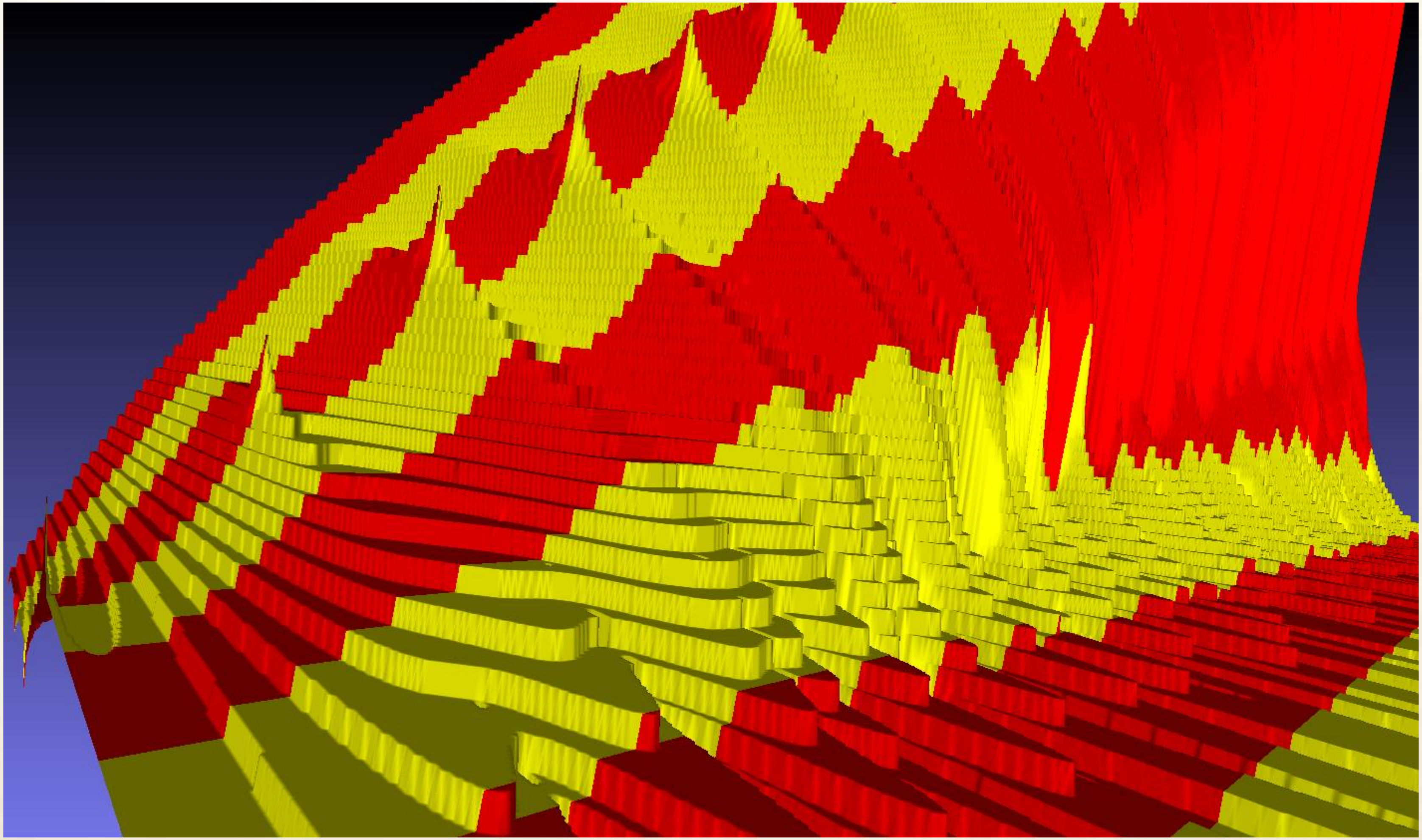
Algoritmo sudėtingumo vizualizacija (1)



Algoritmo sudėtingumo vizualizacija (2)



Algoritmo sudėtingumo vizualizacija (3)



Rymano funkcionalas ir Lancos aproksimacija

$$\zeta(s) = 2^s \pi^{s-1} \ \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\,\zeta(1-s)$$

$$\Gamma(z+1) = \sqrt{2\pi}\left(z+g+\tfrac{1}{2}\right)^{z+\frac{1}{2}} e^{-\left(z+g+\frac{1}{2}\right)} A_g(z)$$

$$A_g(z) = \frac{1}{2}p_0(g) + p_1(g)\frac{z}{z+1} + p_2(g)\frac{z(z-1)}{(z+1)(z+2)} + \cdots.$$

$$A_g(z) = c_0 + \sum_{k=1}^{N} \frac{c_k}{z+k}$$

The coefficients are given by

$$p_k(g) = \sum_{a=0}^{k} C(2k+1, 2a+1)\frac{\sqrt{2}}{\pi}\left(a-\tfrac{1}{2}\right)!\left(a+g+\tfrac{1}{2}\right)^{-\left(a+\frac{1}{2}\right)} e^{a+g+\frac{1}{2}}$$

$$\begin{aligned}
&C(1,1) = 1 \\
&C(2,2) = 1 \\
&C(i,1) = -C(i-2,1) && i = 3,4,\ldots \\
&C(i,j) = 2C(i-1,j-1) && i = j = 3,4,\ldots \\
&C(i,j) = 2C(i-1,j-1) - C(i-2,j) && i > j = 2,3,\ldots
\end{aligned}$$

Rymano-Zygelio formulė (1932 m.)

If *M* and *N* are non-negative integers, then the zeta function is equal to

$$\zeta(s) = \sum_{n=1}^{N} \frac{1}{n^s} + \gamma(1-s) \sum_{n=1}^{M} \frac{1}{n^{1-s}} + R(s)$$

where

$$\gamma(s) = \pi^{\frac{1}{2}-s} \frac{\Gamma\left(\frac{s}{2}\right)}{\Gamma\left(\frac{1}{2}(1-s)\right)}$$

is the factor appearing in the functional equation $\zeta(s) = \gamma(1-s)\,\zeta(1-s)$, and

$$R(s) = \frac{-\Gamma(1-s)}{2\pi i} \int \frac{(-x)^{s-1} e^{-Nx}}{e^x - 1} dx$$

Zetafast algoritmas (2017 m.)

Theorem 1. *Zetafast algorithm for Riemann zeta function:*

Given a positive integer v, $\zeta(s)$ *has for* $\sigma \leq \sigma_0 < v$ *and* $s \notin \{1, \ldots, v-1\}$ *the following representation in terms of three absolutely and uniformly converging series* $D(s)$, $E_1(s)$, *and* $E_{-1}(s)$,

$$\zeta(s) = D(s) + \sum_{\mu=\pm 1} E_\mu(s) - \frac{\Gamma(1-s+v)}{(1-s)\Gamma(v)} N^{1-s} \tag{1.1}$$

$$D(s) = \sum_{n=1}^{\infty} n^{-s} Q\left(v, \frac{n}{N}\right) \tag{1.2}$$

$$E_\mu(s) = (2\pi)^{s-1} \Gamma(1-s)\, e^{i\mu\frac{\pi}{2}(1-s)} \sum_{m=1}^{\infty} E_\mu(m,s) \tag{1.3}$$

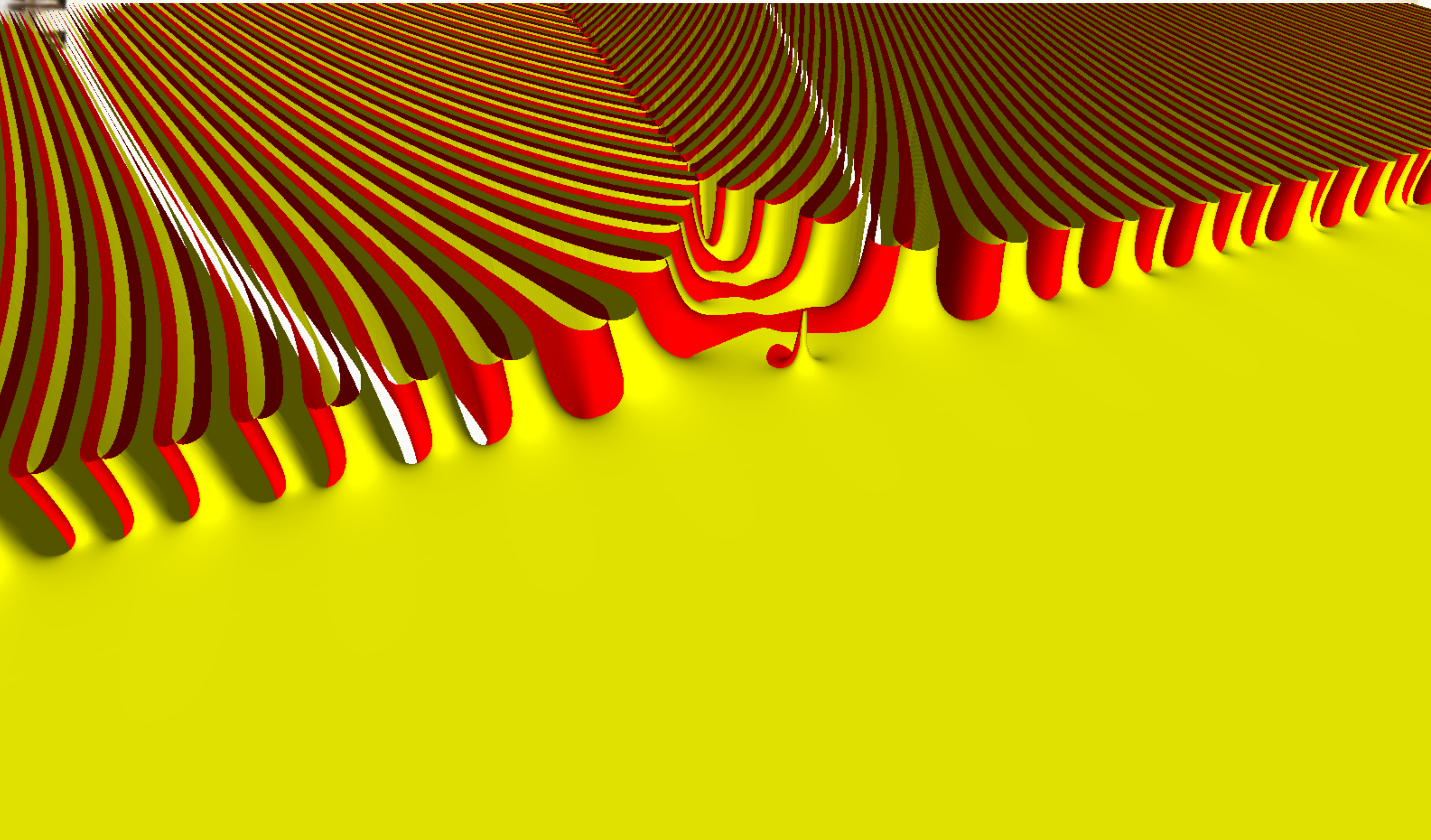
$$E_\mu(m,s) = m^{s-1} - \sum_{w=0}^{v-1} \binom{s-1}{w} \left(m + \frac{i\mu}{2\pi N}\right)^{s-1-w} \left(\frac{-i\mu}{2\pi N}\right)^{w} \tag{1.4}$$

where $Q(v,m)$ *is the normalized incomplete gamma function*

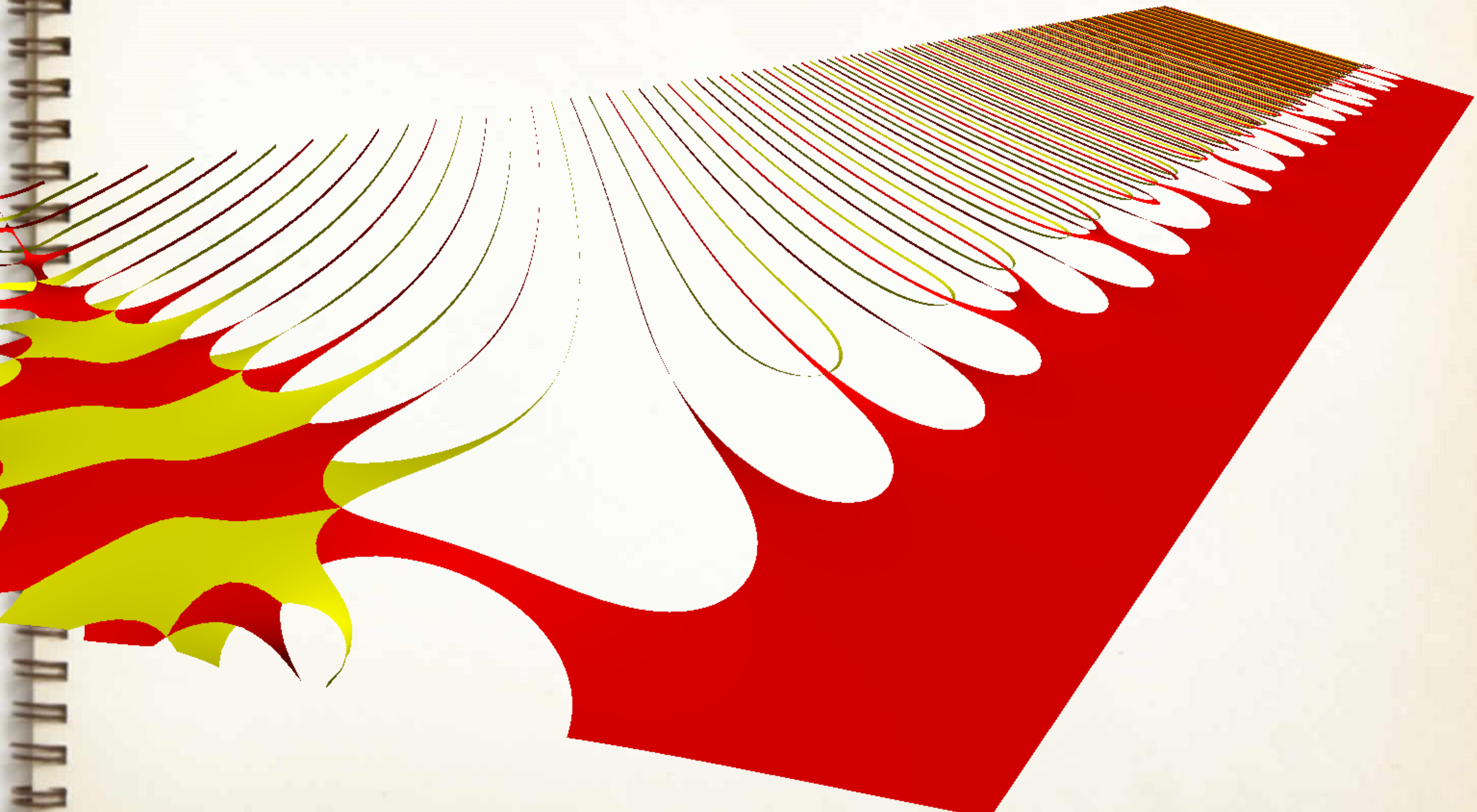
$$Q(v,m) = \sum_{w=0}^{v-1} \frac{m^w}{w!} e^{-m} \tag{1.5}$$

For positive integer $s = k$ *with* $k < v$ *one has to take the limit* $\lim_{s\to k} E_\mu(s)$.

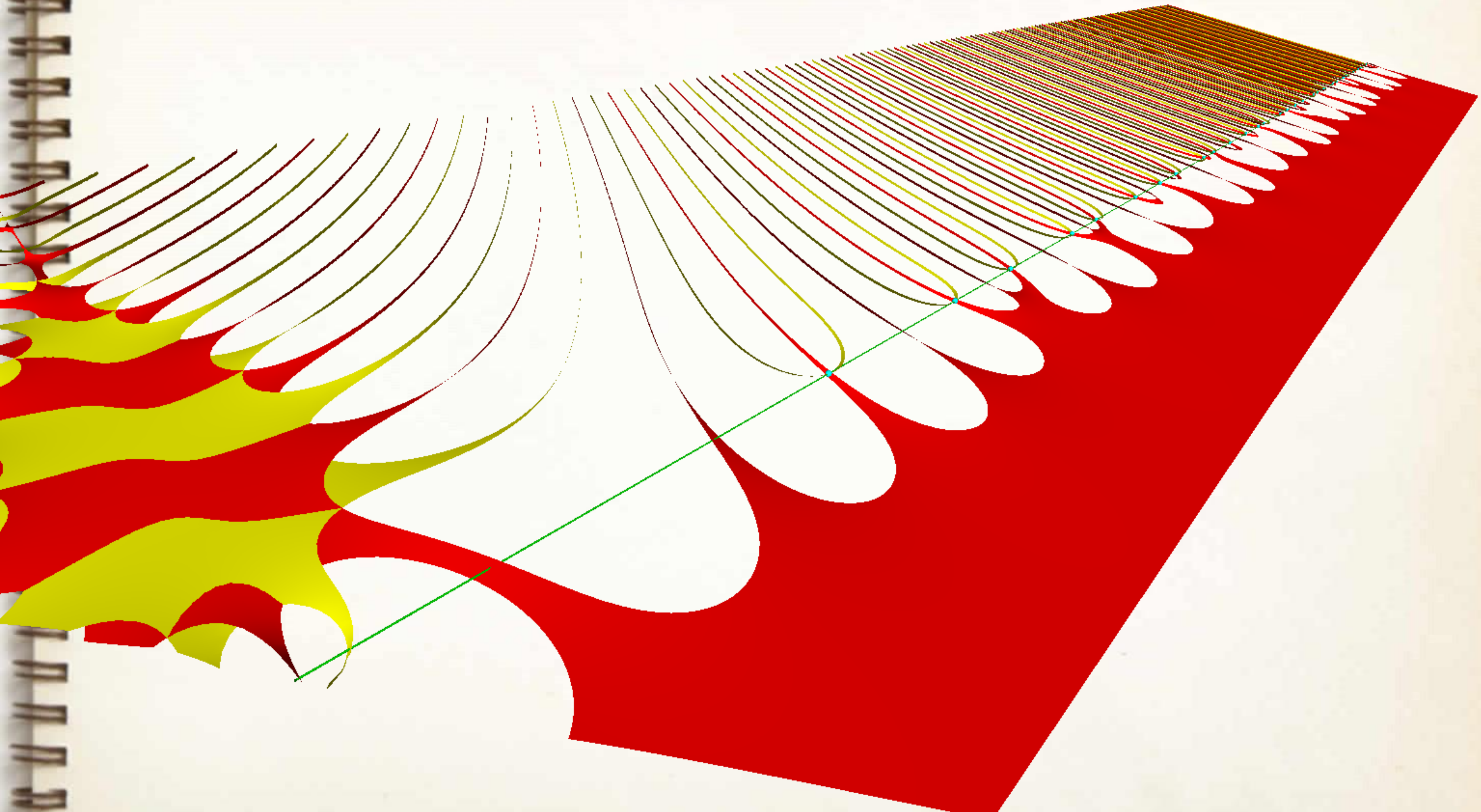
Galutinis vaizdas



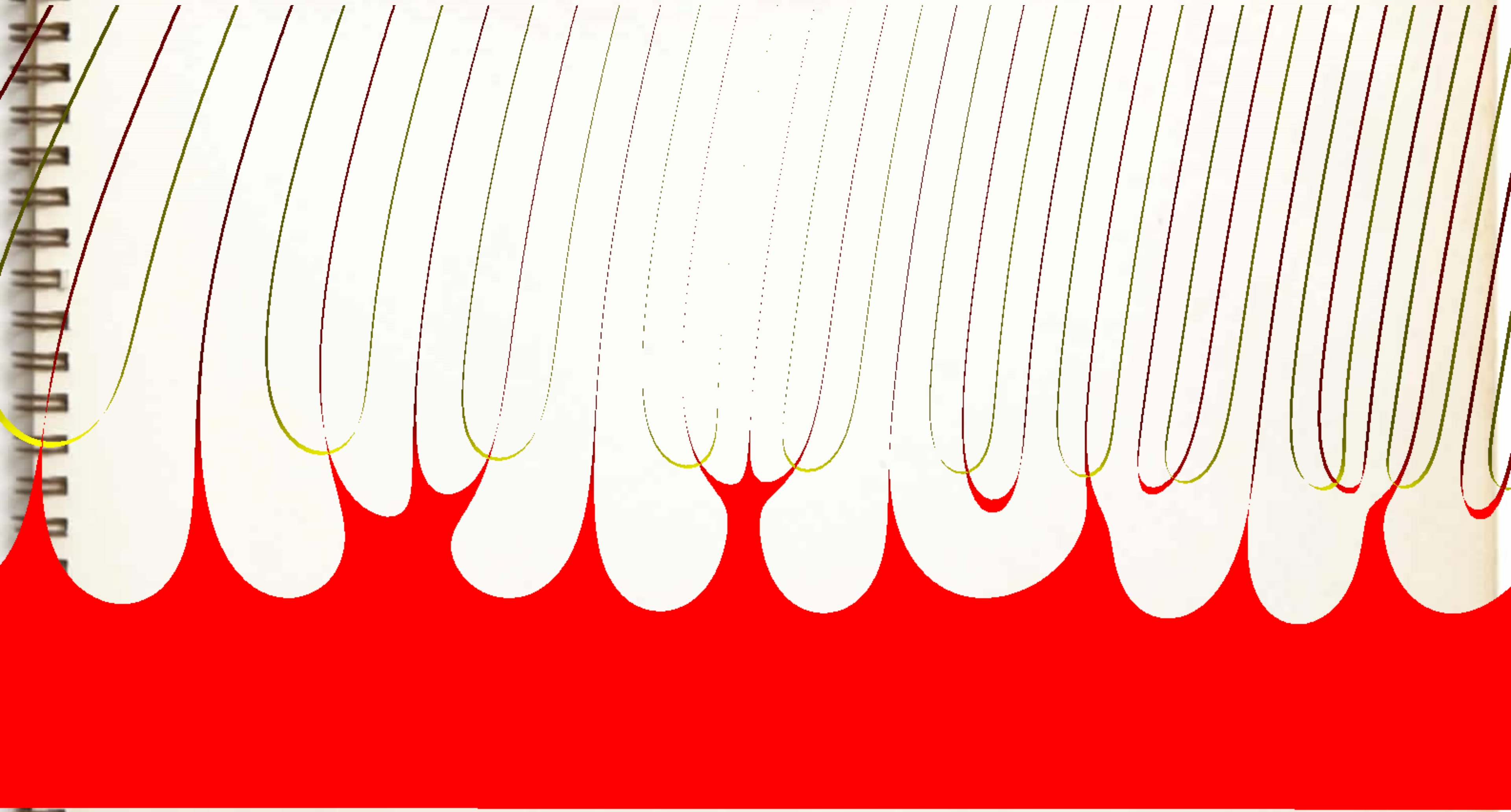
Netrivialūs nuliai (1)



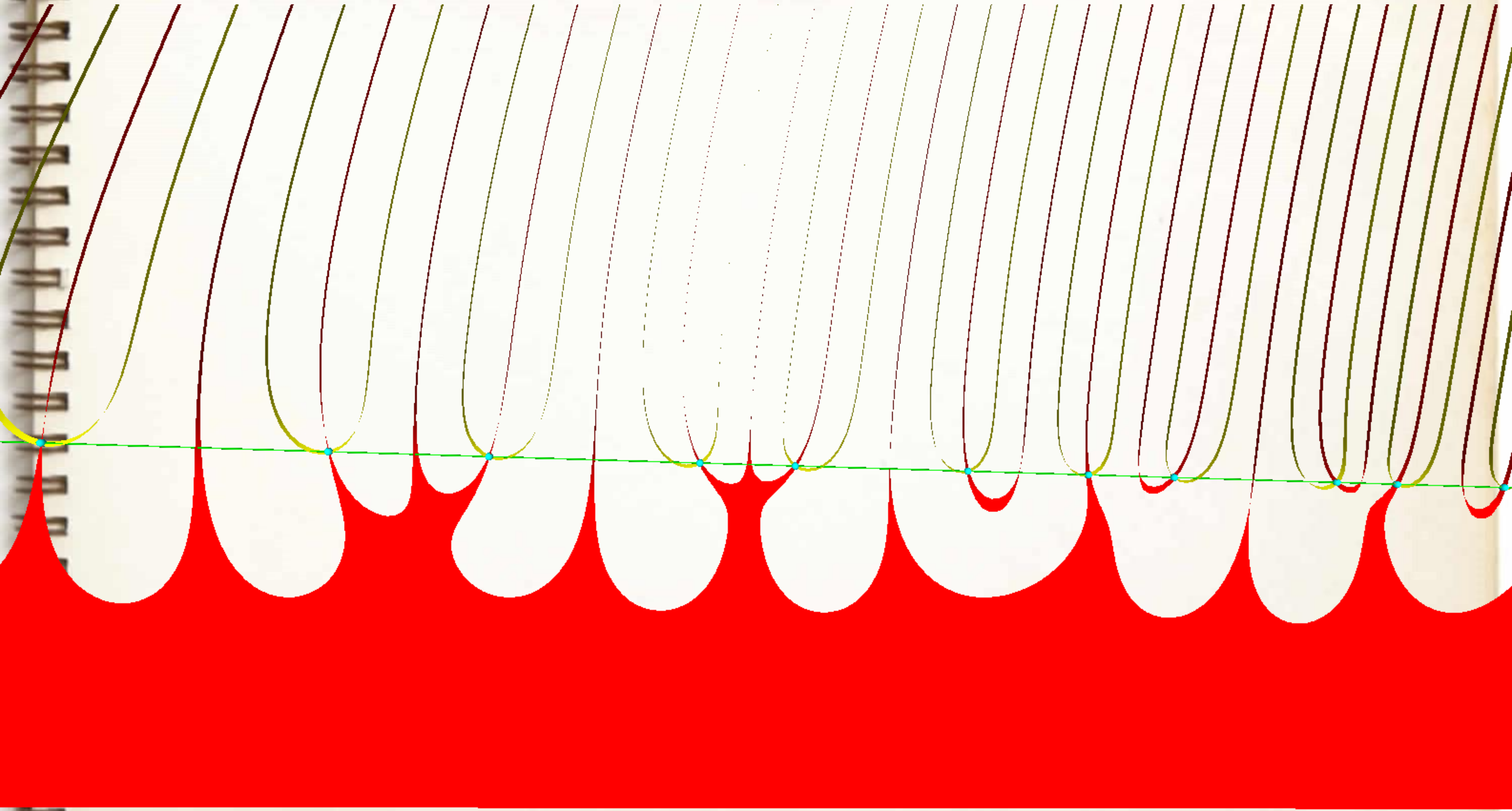
Netrivialūs nuliai (2)



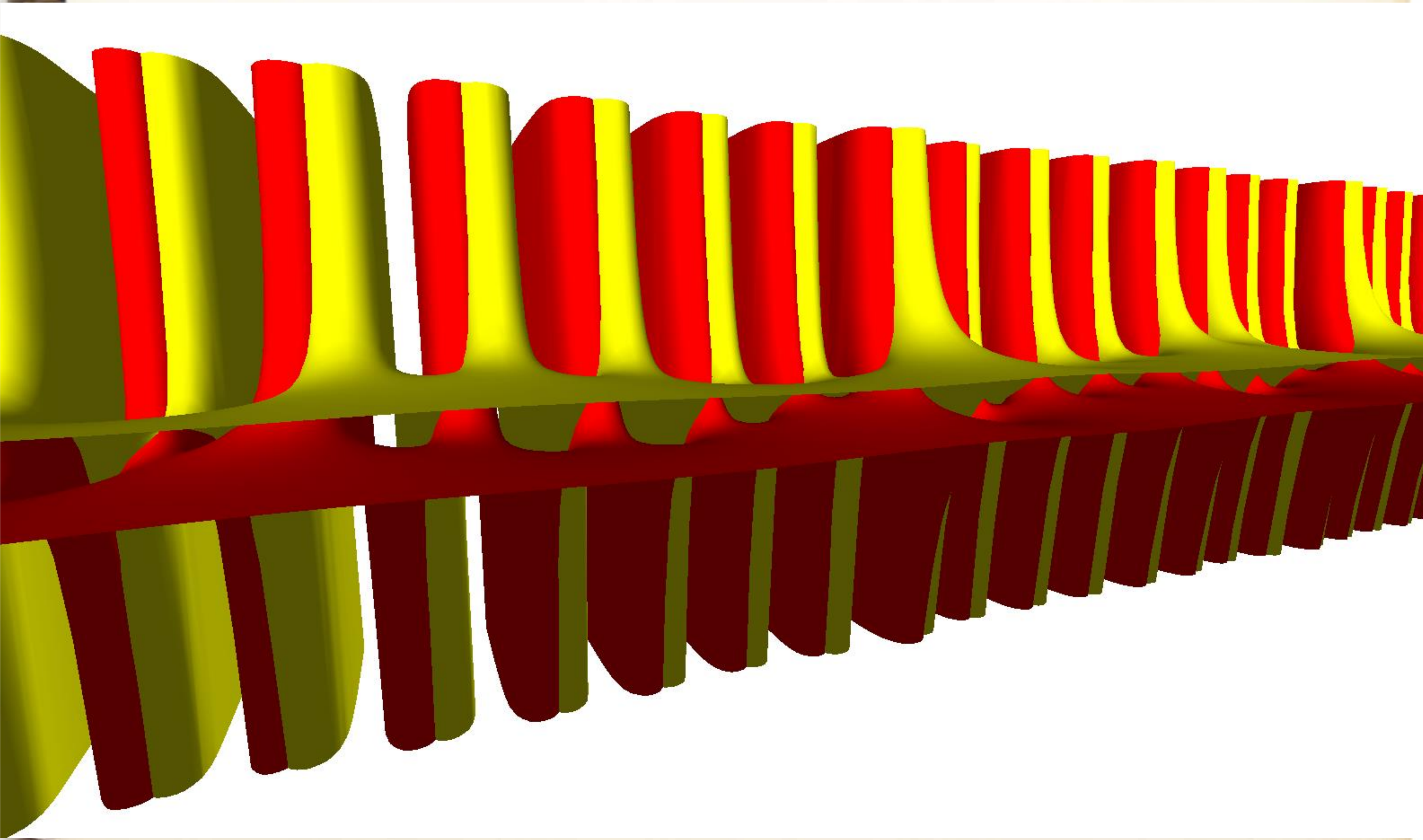
Netrivialūs nuliai (3)



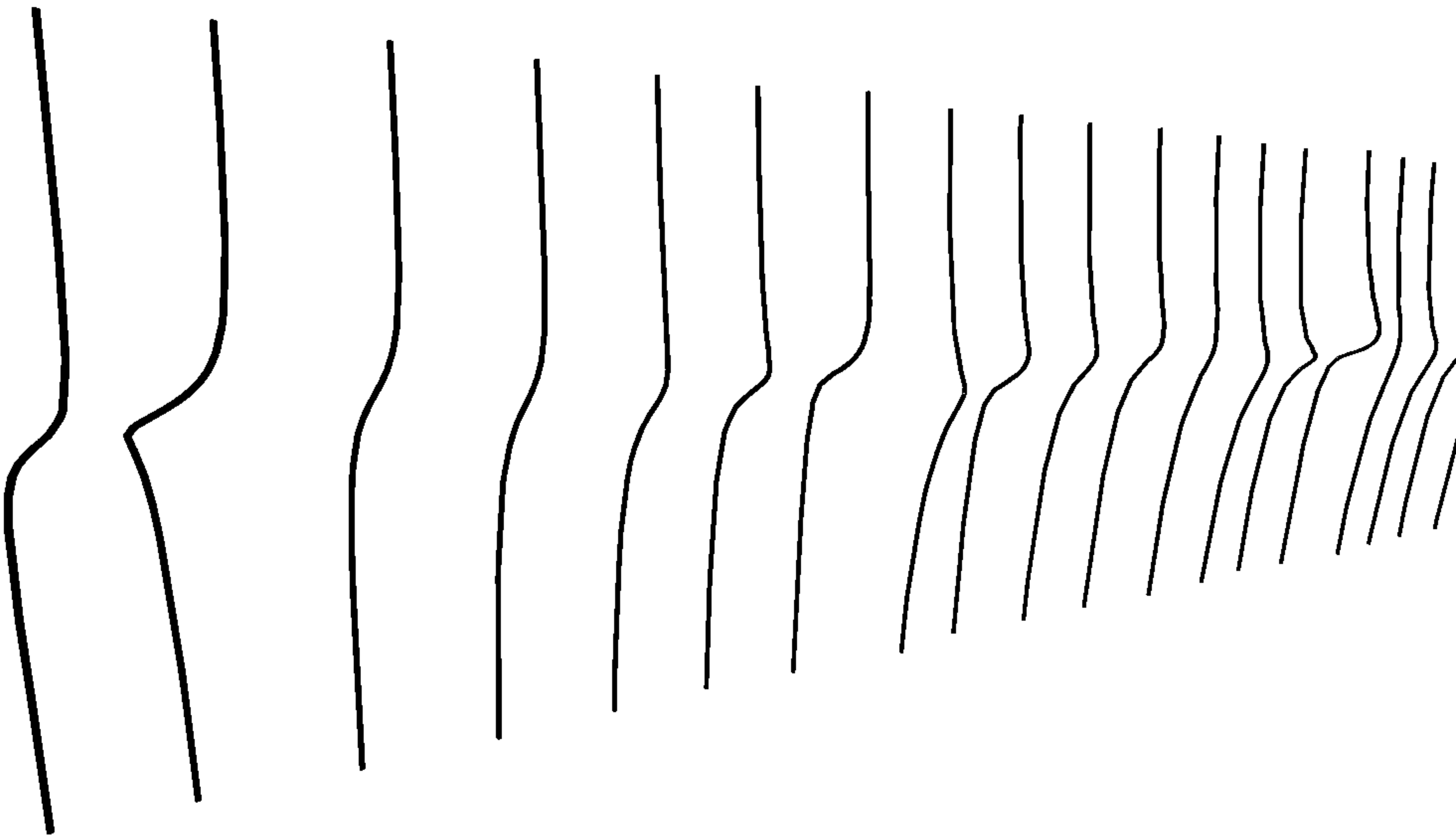
Netrivialūs nuliai (4)



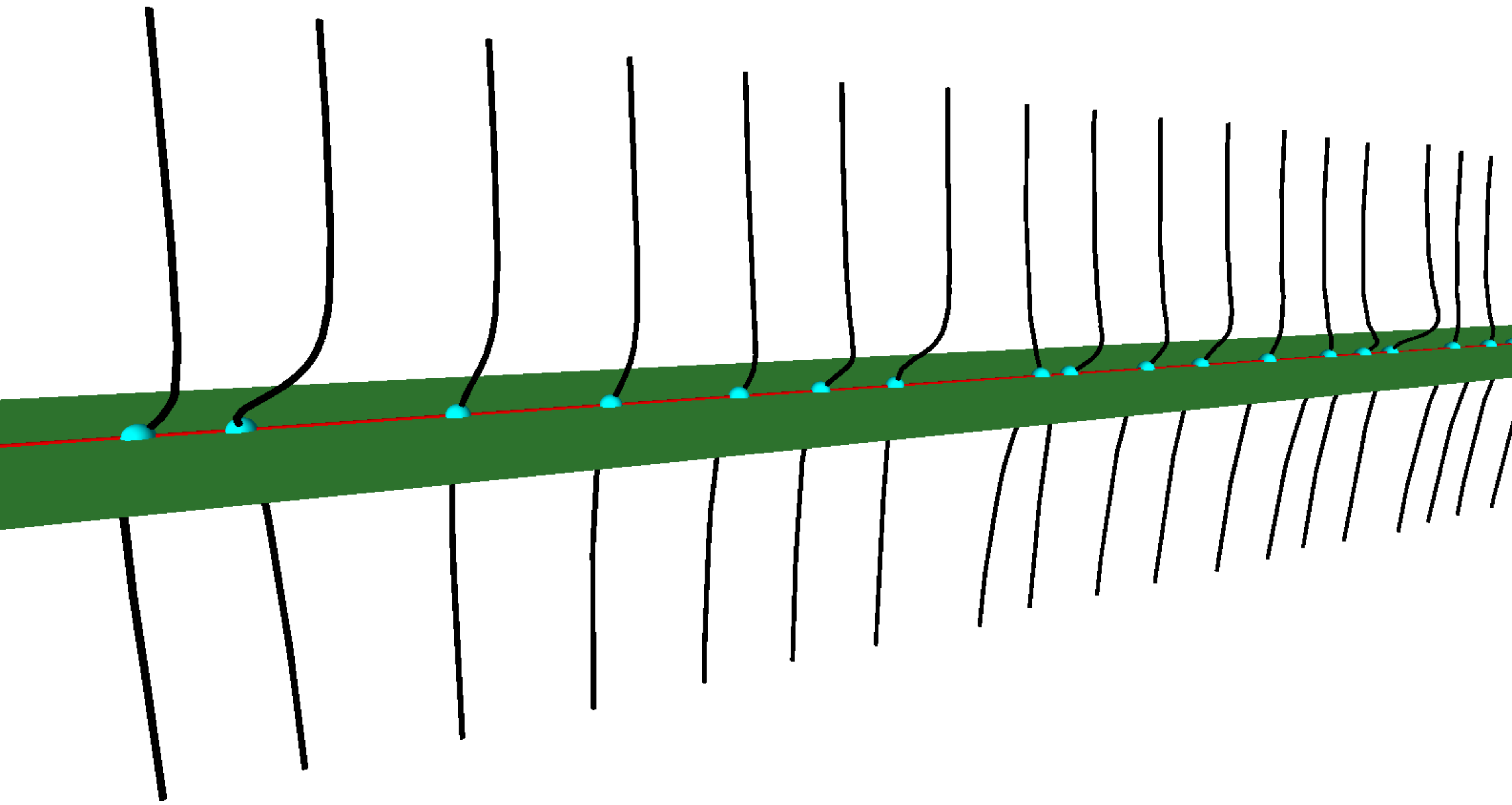
Realusis ir kompleksinis paviršius



Atvaizduojama paviršių sankirta



Nulinės plokštumos kirtimas tiesėje



Ačiū už dėmesį.