DOCTORAL (PHD) STUDIES

COURSE UNIT DESCRIPTION

Course unit title	Scientific areas	Faculty	Institute, department
Image analysis and processing	Informatics engineering (T 007)	Faculty of Mathematics and Informatics	Institute of Data Science and Digital Technologies

Study method	Number of credits	Study method	Number of credits
Lectures	1 (spring)	Consultations	1
Individual works	4	Seminars	1

Summary

During the course, doctoral students will acquire comprehensive knowledge of the mathematical and algorithmic foundations of advanced image processing and classical computer vision. The course bridges digital signal processing, partial differential equations (PDEs), and 3D geometry. To complement data-driven artificial intelligence methods, this module focuses strictly on continuous mathematical modeling, non-linear optimization, and solving inverse problems. Key areas of study include variational methods for image restoration, anisotropic diffusion, advanced multi-resolution and sparse representations (such as Dictionary Learning), and probabilistic graphical models for energy minimization. Additionally, the course deeply explores large-scale 3D scene reconstruction (Structure from Motion, Bundle Adjustment) and covers specific multidimensional applications where mathematical analysis remains indispensable, including medical imaging (Radon transform) and hyperspectral data analysis.

Main Topics:

- Digital Image Fundamentals & Spatial Processing:
  - Continuous vs. discrete image signals. 2D sampling theorem (Nyquist-Shannon) and aliasing.
  - Advanced spatial filtering: Bilateral filtering, guided filters, and Non-local Means (NLM).
- Partial Differential Equations (PDEs) & Variational Methods:
  - The heat equation and isotropic diffusion.
  - Anisotropic diffusion (Perona-Malik equation): Edge-preserving smoothing without resolution loss.
  - Variational methods and inverse problems: Image restoration as an optimization task. Total Variation (TV) regularization (ROF model) for denoising.
- Frequency Domain, Multi-resolution & Sparse Analysis:
  - 2D Fourier Transforms (Continuous and Discrete).
  - Advanced wavelet families: Curvelets, Ridgelets, and Contourlets for specialized line and curve detection.
  - Dictionary Learning and Sparse Coding: The K-SVD algorithm and its application in image inpainting and denoising without neural networks.
- Advanced Segmentation & Graphical Models:

- Active Contours (Snakes) and Level Set methods for segmenting complex, topologically changing objects.
- Probabilistic graphical models: Markov Random Fields (MRF) and Conditional Random Fields (CRF) for modeling pixel interdependencies.
- Energy minimization frameworks: Optimal segmentation and stereo depth mapping using Graph Cuts and Belief Propagation.

- Deep 3D Geometry & Camera Networks:
  - Projective geometry, epipolar geometry, fundamental and essential matrices.
  - Structure from Motion (SfM): Reconstructing 3D space from large sets of 2D images using SIFT/SURF.
  - Bundle Adjustment: Non-linear optimization (Levenberg-Marquardt) to minimize 3D reconstruction errors.
  - Robust estimation: Managing noisy data with M-estimators and advanced RANSAC variants.
- Niche Applications & Multidimensional Analysis:
  - Medical Imaging (CT/MRI): The Radon Transform and Filtered Back Projection algorithms.
  - Hyperspectral and satellite image analysis: Dimensionality reduction using Principal Component Analysis (PCA) for very high-dimensional visual data.

Practical task: students will be asked to solve the specified tasks using image analysis and processing methods.

Main literature

1. Szeliski, R. (2022). Computer Vision: Algorithms and Applications (2nd ed.). Springer. https://szeliski.org/Book/
2. Bredies, K., & Lorenz, D. (2018). Mathematical Image Processing. Springer. https://link.springer.com/book/10.1007/978-3-030-01458-2
3. Gonzales, R. C., & Woods, R. E. (2018). Digital Image Processing (4th ed.). Pearson. http://www.imageprocessingplace.com/
4. Hartley, R., & Zisserman, A. (2004). Multiple View Geometry in Computer Vision (2nd ed.). Cambridge University Press. https://www.robots.ox.ac.uk/~vgg/hzbook/
5. Elad, M. (2010). Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing. Springer. https://link.springer.com/book/10.1007/978-1-4419-7011-4
6. Mallat, S. (2008). A Wavelet Tour of Signal Processing: The Sparse Way (3rd ed.). Academic Press. https://www.di.ens.fr/~mallat/papiers/WaveletTour.html
7. Aubert, G., & Kornprobst, P. (2006). Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations (2nd ed.). Springer. https://link.springer.com/book/10.1007/978-0-387-44588-5
8. Suetens, P. (2017). Fundamentals of Medical Imaging (3rd ed.). Cambridge University Press. https://doi.org/10.1017/9781316671849
9. Schönberger, J. L., & Frahm, J.-M. (2016). Structure-from-Motion Revisited. In CVPR (pp. 4104-4113). https://openaccess.thecvf.com/content_cvpr_2016/html/Schonberger_Structure-From-Motion_Revisited_CVPR_2016_paper.html
10. Bioucas-Dias, J. M., et al. (2013). Hyperspectral remote sensing data analysis and future challenges. IEEE Geoscience and remote sensing magazine, 1(2), 6-36. https://ieeexplore.ieee.org/document/6522459

Lecturer(s) (name, surname)	Science degree	Main publications
Povilas Treigys	Dr.	http://www.elaba.mb.vu.lt/dmsti/?aut=Povilas+Treigys
Jolita Bernatavičienė	Dr.	http://www.elaba.mb.vu.lt/dmsti/?aut=Jolita+Bernatavi%C4%8Dien%C4%97
Gintautas Tamulevičius	Dr.	http://www.elaba.mb.vu.lt/dmsti/?aut=Gintautas+Tamulevičius