On the Arithmetic Optimization Algorithm for the Quadratic Assignment Problem

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Fig 1. Nature-Inspired Heuristic Algorithms — Optimization in Nature's Ways [created by AI]

The arithmetic optimization algorithm is based on using four basic arithmetic operations (+, -, *, ÷).

— The exploitation (intensification) of the search process is achieved by using high-density operators (addition and subtraction).

— The exploration (diversification) of the search process is achieved by using high-dispersion operators (multiplication and division).

THE AOA ALGORITHM. PSEUDOCODE

Algorithm 1 Pseudocode of the arithmetic optimization algorithm (AOA)

```
Arithmetic_Optimization_Algorithm;
// input: Pop_Size – population size, D – (dimensionality) number of components of solutions,
//        Iter_N – the number of iterations of AOA,
//        LB, UB, ε1, ε2, Min_Value, Max_Value, α, μ – parameters (coefficients) of AOA
// output: best_x – the best found solution
01 begin
02   create the initial population of solutions X;
03   C_Iter ← 0;
04   while C_Iter < Iter_N do begin // main loop
05     for i ← 1 to Pop_Size do begin
06       evaluate the objective function for the given solution;
07       memorize the best found solution as best_x
08     endfor;
09     update the MOA value;
10     update the MOP value;
11     for i ← 1 to Pop_Size do // iterate over solutions of the current
                                // population
12       for j ← 1 to D do begin // iterate over components of the current
                                 // solution
13         generate random numbers r1, r2, r3, r4, r5;
14         if r1 > MOA then begin // exploration phase
15           if r2 > 0.5 then // division
16              update the ith solution's component using
                division (÷) operation
17           else // multiplication
18              update the ith solution's component using
                multiplication (*) operation
19         endthen
20         else begin // exploitation phase
21           if r3 > 0.5 then // subtraction
22              update the ith solution's component using
                subtraction (–) operation
23           else // addition
24              update the ith solution's component using
                addition (+) operation
25         endelse
26       endfor; //j
27     // endfor // i
28     C_Iter ← C_Iter + 1 // go to the next iteration
29   endwhile;
30   return best_x
31 end.
```

CASE STUDY: THE QUADRATIC ASSIGNMENT PROBLEM (QAP)

QAP: the goal is to find a (globally optimal) solution (permutation) p=(p(1), p(2),…,p(n)), such that the following objective function is minimized (among all permutations for given matrices A, B):

$$z(p) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} b_{p(i)p(j)}$$

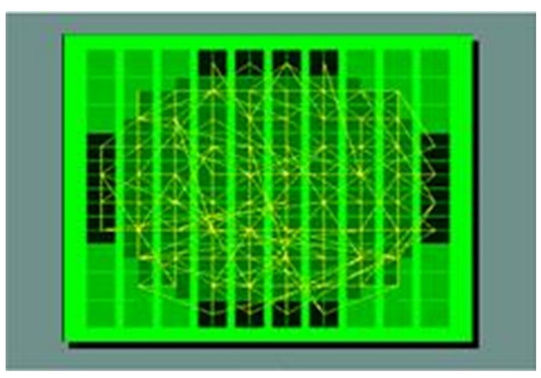

Fig. 2 Illustration of an application of the QAP

(HYBRIDIZED) AOA FOR THE QAP: MAIN COMPUTATIONAL STEPS

```
Begin
  Create the initial population of AOA solutions
  Get QAP solutions using Forward Transformation
  Improve QAP solutions
  Memorize the best QAP solution
  Repeat
    For each population member do
      Generate a new AOA solution
      Get QAP solution using Forward Transformation
      Improve the QAP solution
      Get AOA solution using Backward Transformation
      Update (replace) the population member (AOA solution)
      Update the best QAP solution (if improved)
    Endfor
  Until a termination criterion is satisfied
End
```

(HYBRIDIZED) AOA FOR THE QAP: MAIN CHARACTERISTICS

The main distinguishing characteristics of the proposed algorithm are:

- Bidirectional transformation mechanism: Enables mapping between the continuous search space of the AOA and the discrete permutation space of the QAP, facilitating information exchange between two representations.
- Hybridization with local optimization: Allows hybridization (integration) with local optimizers (such as the tabu search-based algorithms highly specialized to the QAP /Misevicius, 2012/, /Misevičius, Verenė, 2021/, /Misevičius et al., 2024/).
- Synergy of exploration and exploitation: Combines diversified global exploration using arithmetic operators with intensified local exploitation, promoting effective search across different regions of the solution space.

COMPUTATIONAL EXPERIMENTS: BENCHMARK QAP INSTANCES

The computational experiments were conducted using a set of challenging, internationally recognized benchmark instances of the quadratic assignment: tai40a, tai50a, tai60a, tai80a, tai100a, tai27e1—tai27e20, tai45e1—tai45e20, tai75e1—tai75e20, tai125e1—tai125e20, tai175e1—tai175e20, tai343e1—tai343e20.

These instances are from well-established benchmark collections widely used by the international optimization research community: QAPLIB (/Burkard et al., 1997/), Drezner et al., 2005 (/Drezner et al., 2005/), Taillard's QAP benchmark collection (Taillard – QAP (http://mistic.heig-vd.ch/taillard/problemes.dir/qap.dir/qap.html))

As the performance criteria of the algorithm, we adopt the average relative percentage deviation from the best known ((pseudo-) optimal) solutions ($\bar{\theta}$) and the average CPU time per independent run of the algorithm.

ILLUSTRATION OF SELECTED EXPERIMENTAL RESULTS

Instance	Best known value	$\bar{\theta}$	Time (sec)
tai40a	3139370	0.000	841.400
tai50a	4938796	0.000	2650.000
tai60a	7205962	0.000	8735.000
tai80a	13499184	0.119	65720.000
tai100a	21043560	0.153	98430.000
tai27e1	2558	0.000	0.107
tai27e2	2850	0.000	0.094
tai27e3	3258	0.000	0.064
tai27e4	2822	0.000	0.067
tai27e5	3074	0.000	0.058
tai27e6	2814	0.000	0.073
tai27e7	3428	0.000	0.060
tai27e8	2430	0.000	0.096
tai27e9	2902	0.000	0.072
tai27e10	2994	0.000	0.049
tai45e1	6412	0.000	0.569
tai45e2	5734	0.000	0.450
tai45e3	7438	0.000	0.678
tai45e4	6698	0.000	0.534
tai45e5	7274	0.000	0.482
tai45e6	6612	0.000	0.622
tai45e7	7526	0.000	0.535
tai75e1	14488	0.000	5.507
tai75e2	14444	0.000	5.912
tai75e3	14154	0.000	5.540
tai75e4	13694	0.000	5.221
tai75e5	12884	0.000	5.613
tai75e6	12534	0.000	5.944
tai75e7	13782	0.000	14.050
tai125e1	35426	0.000	570.400
tai125e2	36178	0.000	441.800
tai125e3	30498	0.000	583.200
tai125e4	33084	0.000	821.800
tai125e5	37210	0.000	851.000
tai125e6	34624	0.000	911.400
tai125e7	**31020***	0.000	1122.600
tai175e1	57540	0.000	32880.000
tai175e2	**49938***	0.000	30240.000
tai175e3	53900	0.000	26470.000
tai175e4	**60380***	0.000	31220.000
tai175e5	50004	0.000	25430.000

BREAKTHROUGH RESULTS: NEW BEST KNOWN SOLUTIONS

Extensive computational experiments with the proposed hybrid AOA have yielded 25 new best-known solutions for computationally challenging QAP benchmark instances, including large-scale instances with up to 343 facilities.

The new record solutions are for these instances:

tai125e7, tai125e11, tai125e13, tai125e17 (n=125), tai175e2, tai175e4, tai175e7, tai175e8, tai175e9, tai175e10, tai175e12, tai175e13, tai175e17, tai175e19 (n=175), tai343e1, tai343e2, tai343e3, tai343e4, tai343e5, tai343e7, tai343e9, tai343e11, tai343e15, tai343e17, tai343e19 (n=343).

CONCLUDING REMARKS. RECOMMENDATIONS

— The high solution quality achieved confirms the potential and effectiveness of the arithmetic optimization algorithm for solving the QAP.

— The proposed bidirectional transformation mechanism enables effective interaction between continuous and discrete search spaces, contributing to the overall performance of the algorithm.

— Integrating local booster (tabu search based algorithm) accelerates convergence and enhances the capability to tackle large-scale QAP instances.

FUTURE RESEARCH DIRECTIONS

— Comparative evaluation of the performance of other nature-inspired algorithms for the QAP.

— Adaptation and generalization of the bidirectional transformation mechanism to other combinatorial optimization problems.

— Integration of new advanced exploration strategies (such as quantum-inspired mutation processes).

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