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Financing Green Solutions:
Asset Returns and Tail Risks

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Introduction

Scaling climate innovation from laboratory breakthroughs to industrial deployment requires resilient capital flows. In practice, sustainable instruments remain intertwined with conventional markets: if investors perceive hidden contagion, risk premia rise and liquidity dries up during stress. While EU SFDR disclosure rules and Climate Bonds Initiative standards curb greenwashing, the decoupling hypothesis asks whether green assets genuinely diverge from fossil-fuel and broad equity cycles [1].

We study daily returns on green bonds (BGRN), clean-energy equities (ICLN), solar equities (TAN) and EU emissions allowances (EUA), alongside the S&P 500, gold, copper and Dutch TTF natural gas. The price sample runs from 27 November 2018 to 21 November 2025. Preliminary portfolios that included Cardano or Bitcoin showed clustering of VaR breaches and were excluded.

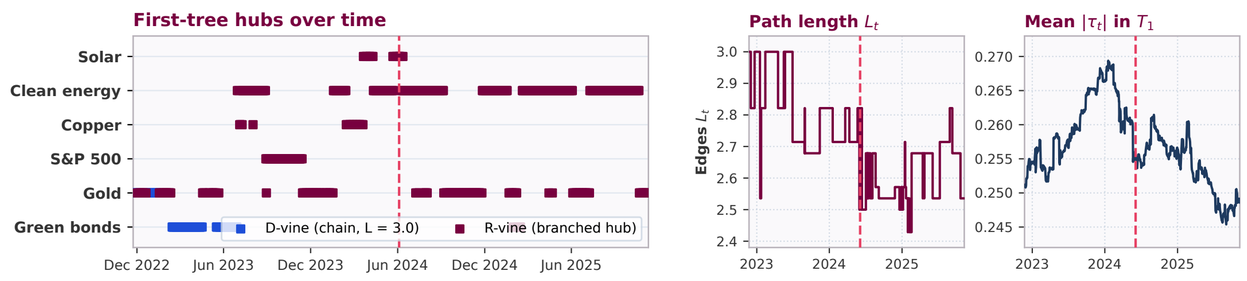
Methodology

We fit an ARMA(1,1) mean with Student-t innovations to each return series. Volatility follows a component GARCH for EU allowances and TTF gas, and a GJR-GARCH for the other six assets. The models use rolling windows of 1,000 observations and are re-estimated every ten observations. For all eight series, Ljung–Box tests on the standardised residuals and their squares and ARCH–LM tests at lag ten gave $p > 0.05$.

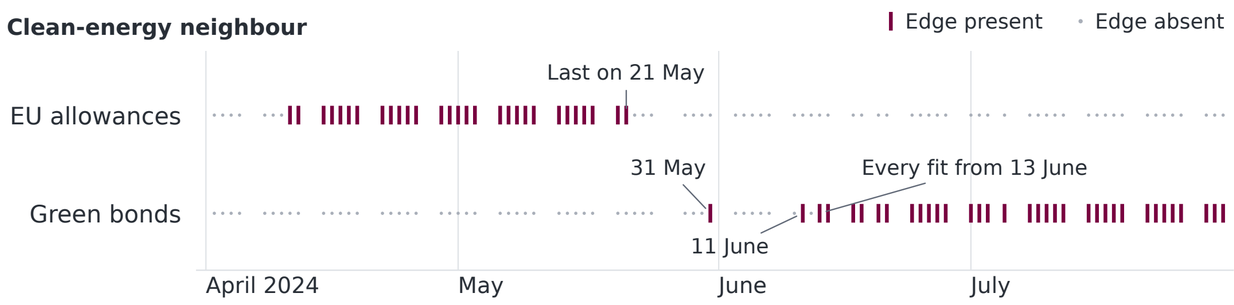
In each window we fit a skewed Student-t distribution to each series of standardised residuals z_j and use its cumulative probabilities u_j as inputs to the vine. The first tree models unconditional pairs. Higher trees model the remaining dependence given the conditioning variables D_e [2, 3].

$$u_j = \widehat{F}_{j,\text{skew-}t}(z_j), \qquad c(\mathbf{u}) = \prod_{m=1}^{7} \prod_{e \in E_m} c_{a_e, b_e | D_e}\left(u_{a_e|D_e}, u_{b_e|D_e}\right).$$

Evidence from the rolling window experiments: dependence structure and backtesting



Across the rolling windows, the panels show which asset has the most neighbours in the first vine tree, the average number of edges separating an asset pair, and the mean absolute Kendalls τ across first-tree edges.



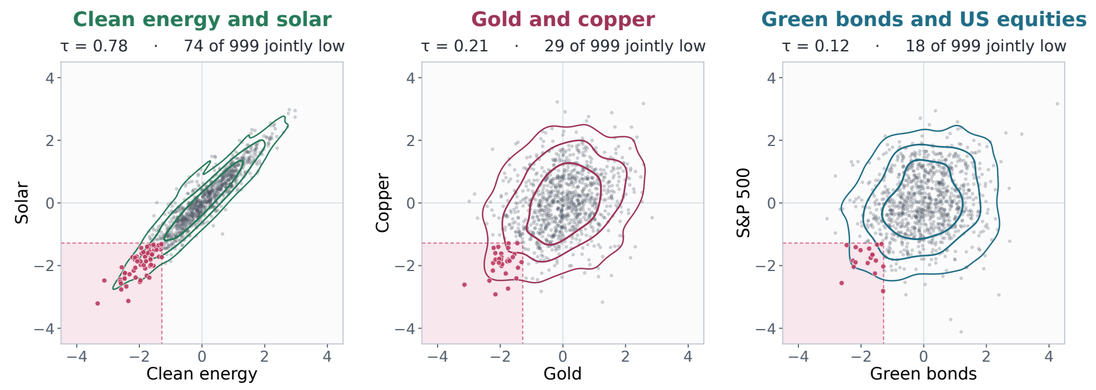
One-day portfolio VaR and ES backtests

		Breach rate		VaR tests (p)			ES tests (p)			
Vine	Level	Expected	Actual	POF	IND	CC	ER	CCL	ESR	Basel
R	99%	.010	.015	.346	.121	.192	.100	.170	.267	Green
R	95%	.050	.056	.464	.096	.192	.069	.171	**.048**	Green
C	99%	.010	.015	.203	.151	.159	.123	.196	.247	Green
C	95%	.050	.055	.569	.079	.182	**.048**	.129	**.037**	Green

VaR tests assess breach frequency (POF), independence (IND) and conditional coverage (CC) [4]. ES tests assess exceedance residuals (ER), conditional calibration (CCL) and expected-shortfall regression (ESR) [5]. Bold values indicate rejection at 5%.

Joint losses and tail dependence

Joint lower-tail observations in the May 2024 window

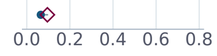


Pink points mark days when both assets fell below their tenth percentiles. On the normal-score scale, the contours enclose 50%, 80% and 95% of the estimated probability mass.

Pair dependence before and after the June transition

Rolling fit dates ● Before 13 June 2024 ◇ From 13 June 2024

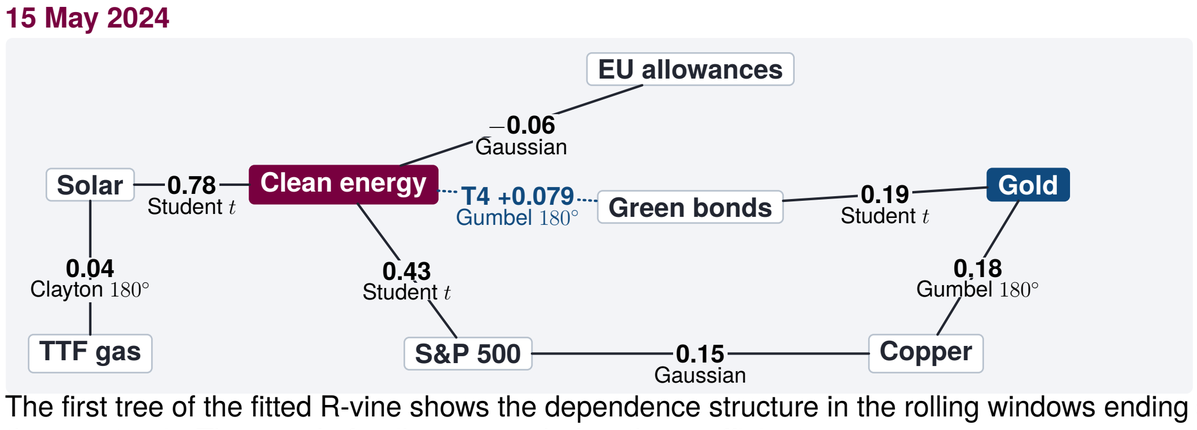
Asset pair	Mean Kendall's τ Before	Mean Kendall's τ From	Lower 5% tail Before	Lower 5% tail From	Upper 5% tail Before	Upper 5% tail From
Clean energy / Solar	0.76	0.77	73%	74%	73%	74%
S&P 500 / Clean energy	0.43	0.40	38%	37%	38%	37%
S&P 500 / Solar	0.37	0.35	33%	32%	33%	33%
Green bonds / Gold	0.20	0.19	20%	18%	20%	18%
Gold / Copper	0.16	0.16	23%	24%	11%	10%
Copper / S&P 500	0.19	0.10	14%	12%	14%	9%
Green bonds / Clean energy	0.09	0.19	12%	19%	11%	19%
Copper / Clean energy	0.16	0.09	15%	10%	11%	9%
Green bonds / Solar	0.08	0.16	11%	17%	10%	16%
Green bonds / S&P 500	**0.07**	**0.15**	**10%**	**15%**	**11%**	**16%**
Copper / Solar	0.14	0.08	14%	9%	10%	8%
Gold / Clean energy	0.09	0.13	11%	11%	12%	14%
Gold / Solar	0.06	0.10	10%	10%	11%	12%



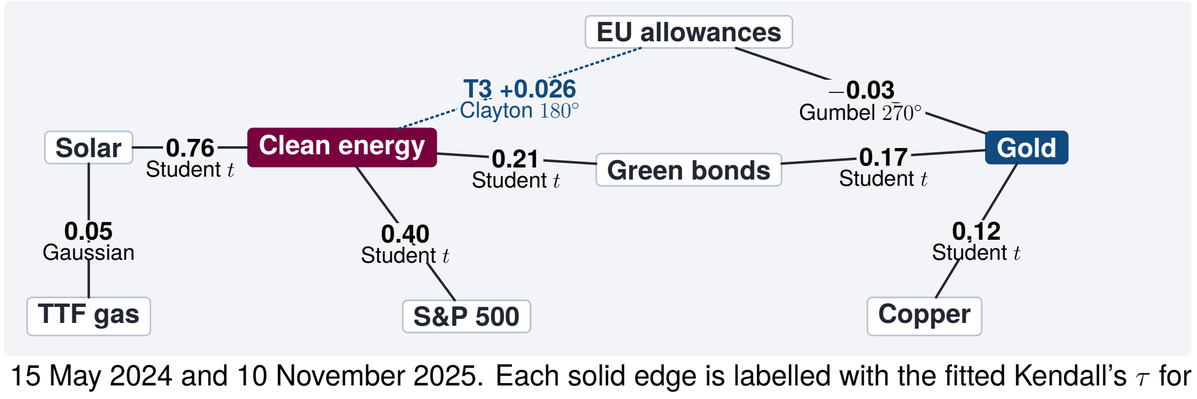
Tail columns show the chance of the second asset entering the same 5% tail as the first, averaged over fits.

First-tree structure before and after June 2024

15 May 2024



10 November 2025



The first tree of the fitted R-vine shows the dependence structure in the rolling windows ending on 15 May 2024 and 10 November 2025. Each solid edge is labelled with the fitted Kendall's τ for the asset pair. The copula family appears below the coefficient.

Pairs that do not share a first-tree edge appear in higher trees, where their dependence is estimated conditional on other assets. The dotted blue edges show two such relationships and identify the corresponding tree. Green bonds and clean energy move from tree 4 in May 2024 to a direct link in November 2025. Over the same period, EU allowances and clean energy move from a direct link to tree 3.

Split decoupling and panic coupling

- Empirical dependence rejects universal decoupling: **clean-energy** (ICLN) and **solar** (TAN) equities move in lockstep ($\bar{\tau} = 0.76$, $\bar{\lambda}_L = \bar{\lambda}_U = 0.74$) with strong S&P 500 integration ($\bar{\tau} = 0.36$–0.42), making ICLN the main bridge for equity shocks.
- **Green bonds** (BGRN) decouple from equities ($\bar{\tau} = 0.11$) and clean tech ($\bar{\tau} = 0.14$) while co-moving modestly with gold ($\bar{\tau} = 0.20$) as fixed-income stabilizers [1].
- While **gold** (GCF) and **copper** (HGF) show weak mean correlation ($\bar{\tau} = 0.16$), lower-tail crash dependence ($\bar{\lambda}_L = 0.23$) exceeds double upper-tail boom dependence ($\bar{\lambda}_U = 0.11$).

Topology shift around June 2024: a new regime?

- A secondary hub around **gold** (GCF) emerges only intermittently via **EU carbon allowances** (EUA): *pro-cyclical industrial demand* and *counter-cyclical EU policy hedging* offset one another, masking structural linkage behind near-zero net rank correlation.
- Once 2020–2022 crisis observations roll out of the rolling window, around June 2024, fossil–green correlations decrease. Topology shows a strong hub around ICLN, systemic lower-tail risk ($\bar{\lambda}_L \approx 0.16$) persists, and shorter paths imply potentially faster shock propagation.

Conclusions & future directions

- **Cardano** and **Bitcoin** failed univariate screening due to extreme volatility and clustered tail breaches; accommodating them requires EVT or similar tail extensions.
- Green assets are moving past their speculative phase, splitting between equity-coupled clean-tech hubs (ICLN, TAN) and decoupled green bond portfolio diversifiers (BGRN).
- Post-June 2024 centralization brings more hubs and faster shock transmission despite lower volatility, motivating direct regime-switching and structural-break models.

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