Ranks of Hankel Matrices in Estimation of Remaining Useful Life of Bearings

P. Sabanskytė[1], M. Landauskas [1]
[1] Kaunas University of Technology, Kaunas, Lithuania
paulina.sabanskyte@ktu.edu, mantas.landauskas@ktu.lt

Motivation

Estimation of the remaining useful life (RUL) of bearings is usually performed using various methods, including machine learning, entropy-based, or other more or less classical methods. This study aims to use the ranks of the associated Hankel matrix.

Algebraic techniques in predictive diagnostics are still less common, although some applications of Hankel matrices, for example, have received more attention recently [2, 3, 4].

Introduction

Our research focuses on analyzing the remaining useful life of bearing based on their vibrational signals. A unique approach to this problem is the introduction of an algebraic techniques (in particular – Hankel matrices) a rather novel idea in predictive diagnostics.

By applying a fixed sliding window to vibration data, we can construct Hankel matrices and find their so called feature – pseudo-rank (Fig. 1). Being a numerical feature, it represents bearings' RUL (or health condition which is essentially the inverse of RUL, depending on what exactly needs to be estimated) at any given time. The question arises during practical applications: how to estimate the pseudo-rank? Thus we introduce a methodology to solve this problem.

Algebraic approach is a topic of the ongoing research and will further be developed for problems in predictive diagnostics.

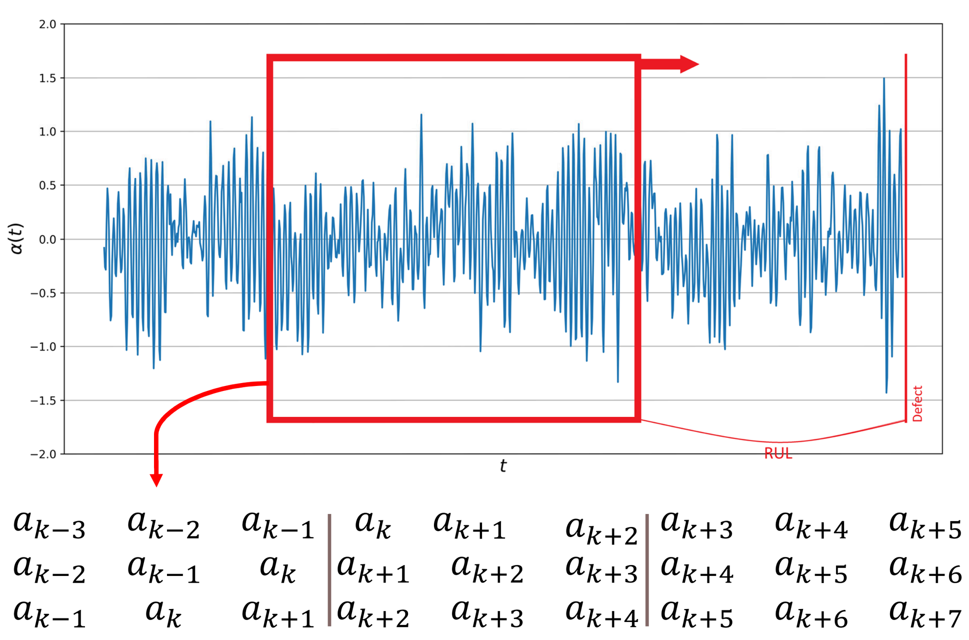

Fig. 1 Moving windows produces subsequences which are then transformed into associated Hankel matrices.

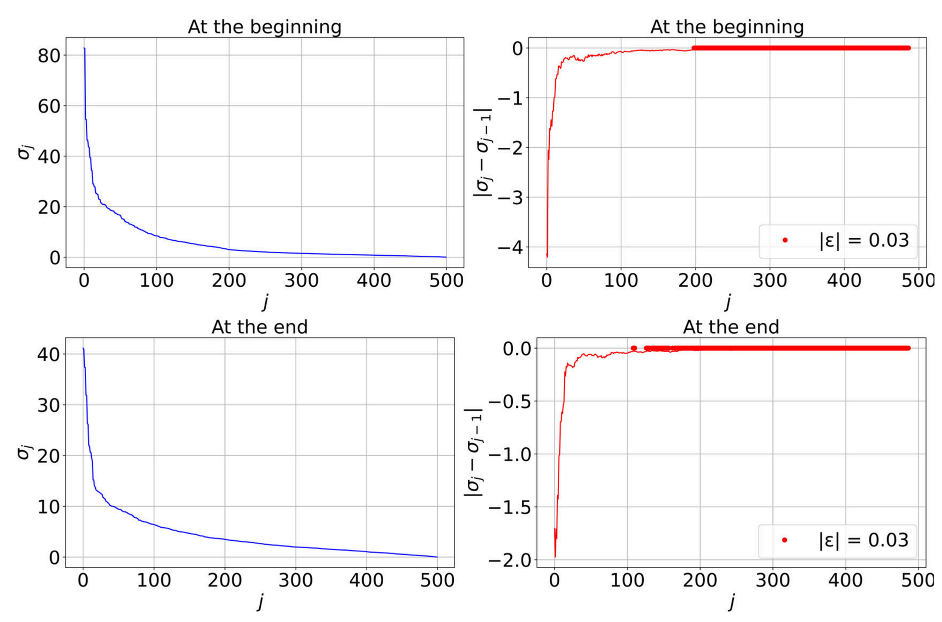

Fig. 2. A threshold on the average differences of adjacent singular values is employed for rank estimation.

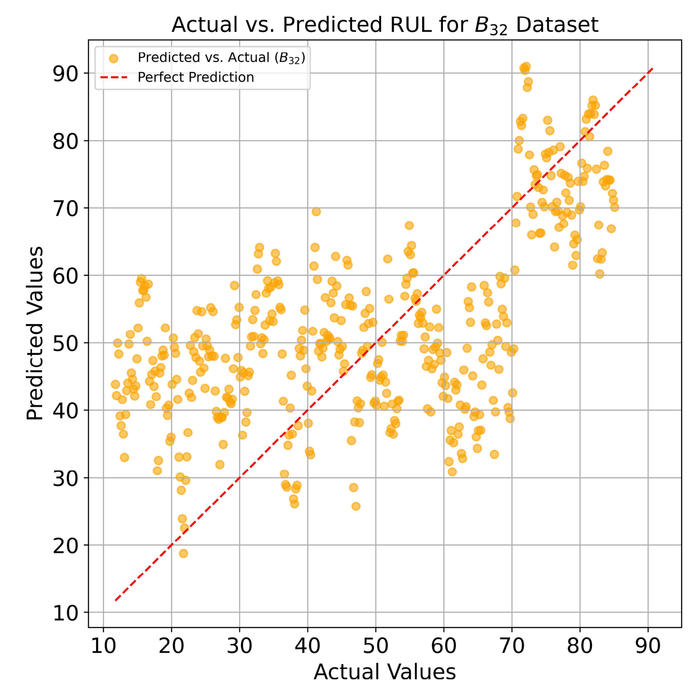

Fig. 3. Prediction of RUL for B_{32} dataset (validation) with the multidimensional model trained on B_{31}.

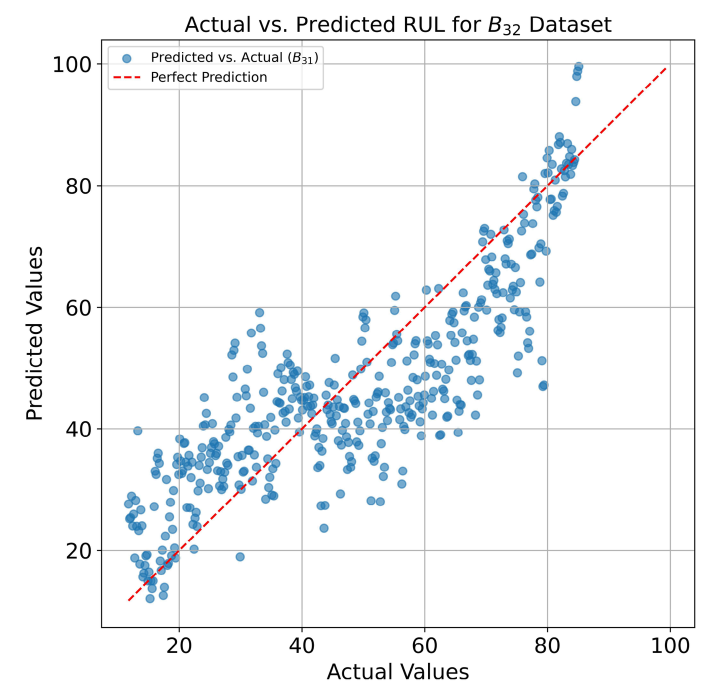

Fig. 4 . Prediction of RUL for B_{31} dataset (training) with the multidimensional model.

Numerical analysis

RUL prediction starts by evaluating bearings vibration at any given time moment. Then, one needs to find the rank of the associated Hankel matrix precise enough to represent an algebraic feature of vibrational data (exact formal ranks of a sequence does not exist due to the presence of noise).

The solution of the rank estimation problem is demonstrated with the data from experimental platform PRONOSTIA [1]. Here, accelerometers measures bearings' accelarations a_x, a_y along Ox and Oy axes respectively. The value of accelaration changes over time and begins to show noticeable increase in standard deviation for small RUL values. The experiment is conducted with custom ball bearings. Conditions of the experiment are: 1500 rpm and 5000 N load.

Since the bearing's data files includes the timestamps of the data registered they are used to determine bearing's RUL. Despite the data being registered in two directions, only the first one (along Ox axis) is considered here.

The feature of interest is the pseudo-rank r of the sequence $\{a_i\}_{i=0}^{n}$. It is usually found by counting the number of singular values σ_j, $j=\overline{1,m}$, which are greater than a threshold ε. Our variation is to compare $\left\{|\sigma_j-\sigma_{j-1}|\right\}_{j=1}^{m}$ to ε instead. The first index j at which the moving average of the difference (of window w) is less than ε three times in a row is considered $r_k = r\left(\{a_i\}_{i=k}^{k+2m-2}\right) := j$ (Fig. 2).

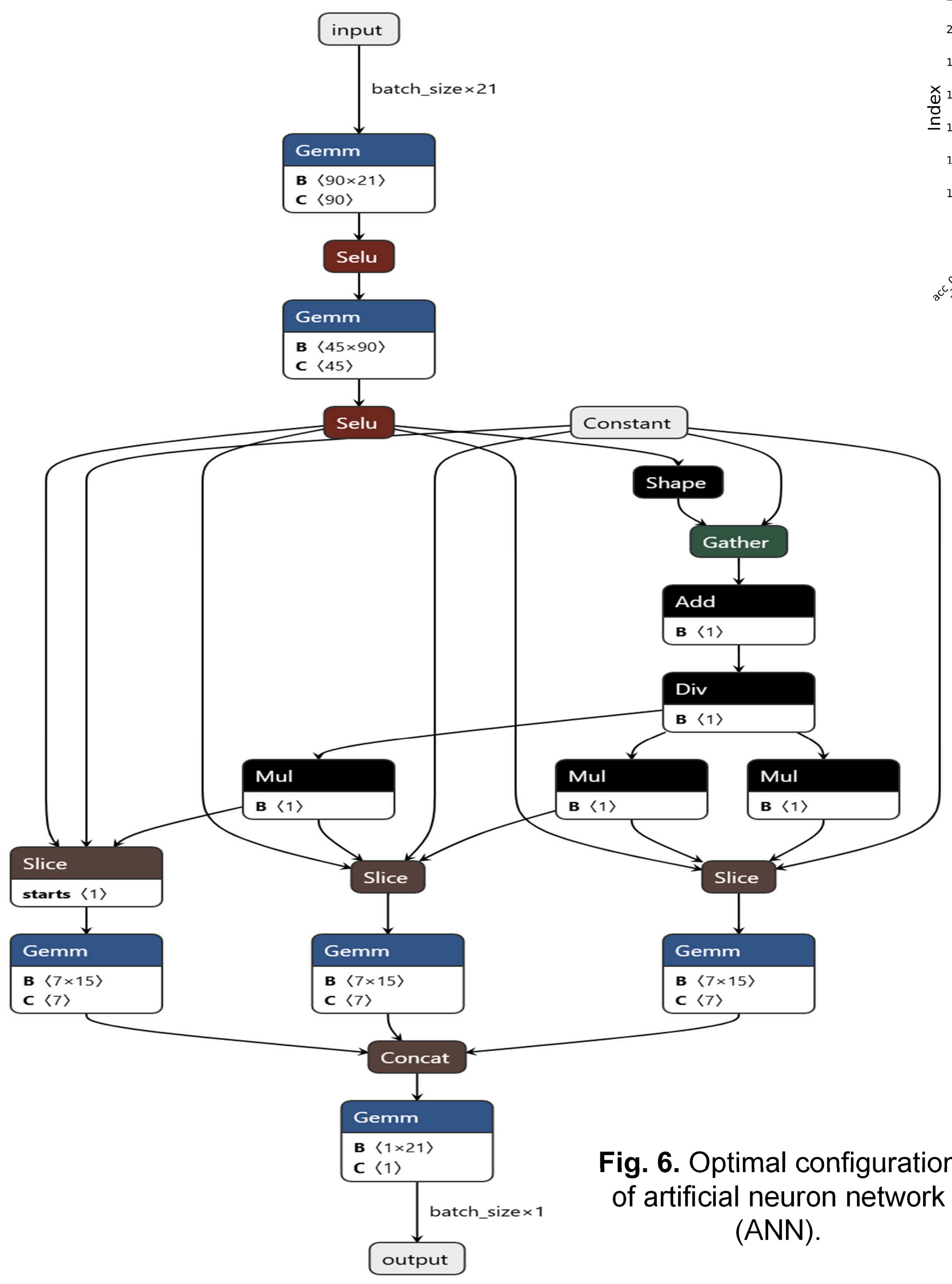

Fig. 6. Optimal configuration of artificial neuron network (ANN).

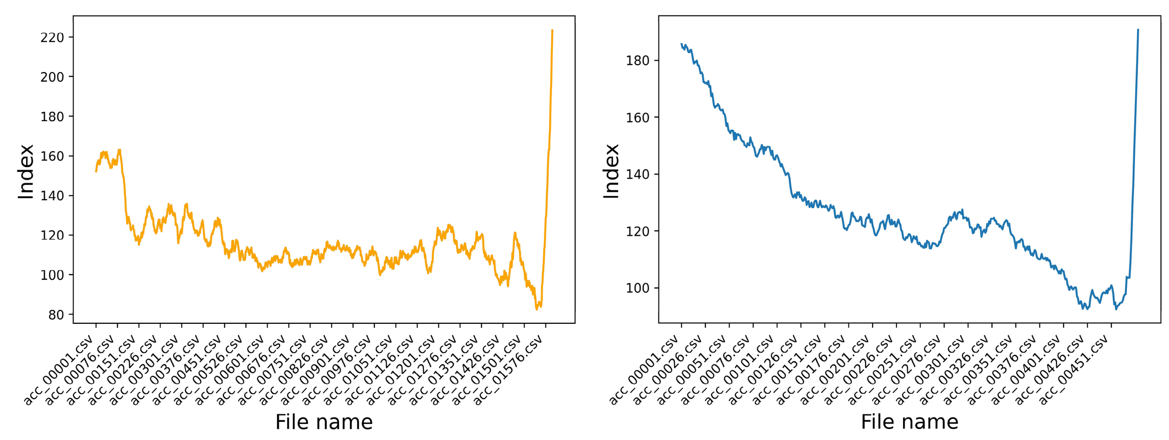

Fig. 5. Evolution of Hankel matrix pseudo-rank values for testing dataset B_{32} (left) and for training dataset B_{31} (right). Optimal parameters $(m, w, e) = (500, 13, 0.03)$.

The optimal values for m, ε, w are determined by particle swarm optimization, minimizing prediction accuracy M.

$$m_{opt}, \varepsilon_{opt}, w_{opt} = \arg\min_{m,\varepsilon,w} M(m,\varepsilon,w)$$

1D model. A perceptron is used (3 hidden layers having 32, 16, 8 neurons respectively).

- Input: r_k.
- Output: $RUL_k = t_{end} - t_k$.

ND model. A custom functional ANN architecture is used.

- Input: $(r_{k-N+1}, r_{k-N+2}, \dots, r_k)$.
- Output: $RUL_k = t_{end} - t_k$.

The target function in both cases is the same:

$$M(m,\varepsilon,w) = MSE\left(RUL_k^{(pred)}, RUL_k^{(actual)}\right)$$

Numerical experiments suggest that in real applications the parameters needs to be optimized for a particular bearing even if experiment parameters are the same.

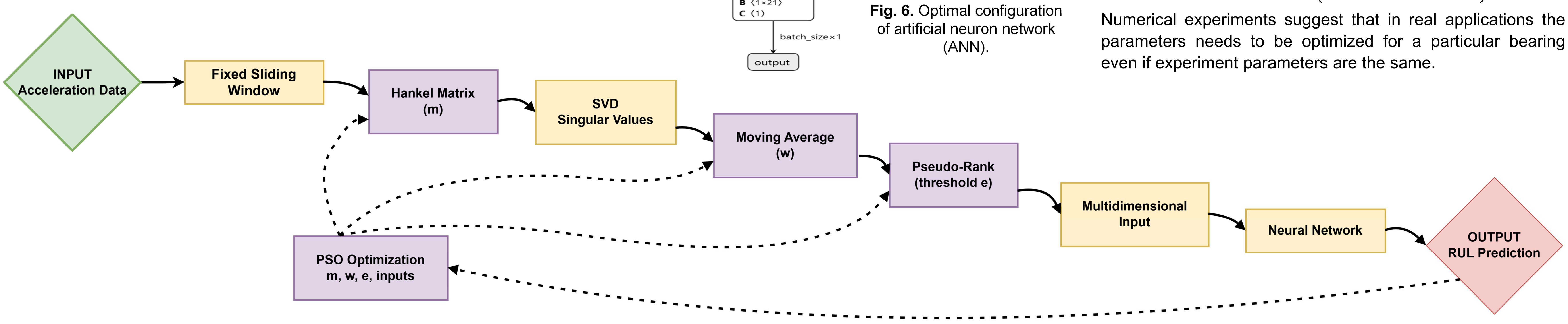

Fig. 7. Complete workflow of the data preprocessing, model optimization and RUL prediction.

Discussion

Prediction of RUL is only viable at the end of bearing's lifetime if 1D model is used. Only then r_k shows a stable tendency to increase over the remaining time (Fig. 5). Successional predictions would be required to detect the decrease in RUL and predict it from that point onward.

Optimal configuration of ANN for ND model was formed by training a number of different arbitrary architectures. The best prediction error was achieved with a network having 3 different "towers" for independent feature assessment (Fig. 6). Heuristic argument for this is that r_k shows decreasing tendency at first but then starts to increase at the end of bearing's lifetime. ND approach is capable of predicting RUL in advance and for longer period of time.

Aknowledgements

Research is supported by the Research Council of Lithuania (LMT grant no. S-ST-25-197).

References

1. Nectoux, P., Gouriveau, R., Medjaher, K., Ramasso, E., Chebel-Morello, B., Zerhouni, N. (2012). PRONOSTIA: An experimental platform for bearings accelerated degradation tests, 1-8.

2. Ojha, S., & Shelke, A. (2025). Estimation of the Remaining Useful Life of Axially Loaded Members Through Nested Least-Square Support Vector Regression. *Journal of Earthquake Engineering, 29 (1)*, 243-264.

3. Fan, W., Jiang, F., Li, Y., & Peng, Z. (2024). A Hankel matrix-based multivariate control chart with shrinkage estimator for condition monitoring of rolling bearings. *IEEE Transactions on Automation Science and Engineering.*

4. Bisoyi, S., Rathi, A. K., & Mahato, S. (2024, November). Improvements in Prediction of Remaining Useful Life of Roller Bearings Using Deep Learning Models. *In 2024 8th International Conference on System Reliability and Safety (ICSRS) (pp. 688-694). IEEE.*