Asymmetric Univariate and Multivariate Financial Time Series Models and Their Applications

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Introduction

One of the main goals in analysis of financial time series is volatility observation. Volatile part of returns is undefined and therefore considered the risky part of an investment. By modelling volatility, a level of risk can be identified. A common approach to analyse volatility is by using a variety of univariate and multivariate GARCH models (Generalized AutoRegressive Conditional Heteroskedasticity) that evaluate risk of a stock through it's own past time information and through inter-relations between stocks.
Attention is drawn to models that try to better fit asymmetric, leptokurtic data and also better explain the volatile part of the price by introducing external variables.

Goals

- Evaluate and compare risk of stocks.
- Verify the importance of asymmetry, heavy-tailed distribution and external regressors inclusion in risk evaluation.

Data

Stock price of 4 European airline companies: Ryanair, Wizzair, EasyJet, Norwegian.

Univariate GARCH methods

General GARCH models tend to have such assumptions as symmetry and normal distribution, which might be quite restrictive for the financial time series. Therefore, the use of different asymmetric models are verified together with the observation of different distribution assumption.

1. Asymmetry assumption
   - GJR-GARCH (Glosgen-Jagannathan-Runkle, 1993)
     Asymmetry is achieved by changing the slopes of news impact curves.
     $$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1}) r_{t-1}^2 + \beta \sigma_{t-1}^2,$$
     where $I_{t-1} = 1$, if $r_{t-1} < \mu$, 0 if $r_{t-1} \geq \mu$; γ - asymmetry coefficient.
   - NA-GARCH (Non-linear Asymmetric, 1993)
     Asymmetry is achieved by moving news impact curve to the left or right.
     $$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha \left(r_{t-1} + \gamma (\sigma_{t-1}^2)^{1/2} \right)^2.$$
2. Distribution assumption
   - Normal distribution
   - Heavy-tailed distribution - GED (Generalized Error Distribution)
     $$dF(x|\mu, \sigma, \kappa) = \frac{e^{-\frac{1}{2}\left|\frac{x-\mu}{\sigma}\right|^{\frac{1}{\kappa}}}}{2^{\kappa+1}\sigma\Gamma(\kappa+1)} dx.$$

Empirical research

In order to find the most risky stock and find which distribution and asymmetry modelling are the best fit to the selected data, 4 models were tested, each asymmetric GARCH was assigned two different distribution assumptions.

- AIC and likelihood were used as main diagnostic parameters for model comparison
- Estimated parameters are main indicators for companies' risk comparison

	Distribution		Model			
	Normal	**GED**	**GJR**	**NA**	**Asymmetry**	**Risk order**
Ryanair	×	✓	✓	✓	×	1
Wizzair	×	✓	✓	×	✓	2
EasyJet	×	✓	✓	✓	✓	3
Norwegian	×	✓	✓	×	✓	4

(a)

Figure: Univariate methods comparison
Source: created by the author.

Conclusions:

- Models with heavy-tailed distribution have better diagnostics.
- Asymmetry was found in some companies (Ryanair, Wizzair, EasyJet), depending on model and distribution selected.
- GJR-GARCH is the preferred volatility modelling for the selected data
- Risk order: Ryanair → Wizzair →EasyJet →Norwegian

External Regressors

- Oil price
- Terrorist attacks (dummy variable)
- Hurricane Irma (dummy variable, 2017-09)
- Change points
- News impact

News Impact calculation

Reuters.com news are collected → text mining: unuseful words removed from text → sentiment analysis: words are assigned positive or negative sentiment → total positivity or negativity rate is calculated → bad sentiments have higher impact therefore they are added higher weights → method repeated for each article, each company → daily values are calculated → 4 time series with news values are obtained.

Change points detection

- NP-ICSS (Non-Parametric Iterative Cumulative Sums of Squares, Mood, 1954)
  Statistics are calculated through ranks and compared to the critical value.
- WBS (Wild Binary Segmentation, Korkas and Fryzlewicz, 2014)
  Statistics are calculated through ranks and compared to the critical value.

Monte Carlo simulation performed with predetermined number of change points, assuming normal and heavy-tailed distribution. Conclusions are that WBS show better accuracy especially for bigger samples. Adding non-normality assumption reduces WBS accuracy from 90% to 50%. WBS further used on selected data.

Multivariate GARCH methods

Univariate GARCH is limited to analysing only self-related volatility. Multivariate GARCH models give additional information about inter-relations between companies. Therefore, a few asymmetric multivariate models are observed. Additionally, external regressors inclusion is analysed. Most commonly used multivariate asymmetric GARCH models:

1. mGJR-GARCH (a multivariate extension of GJR-GARCH, McAleer, 2009)
   $$H_t = W + \sum_{l=1}^{r} A_l \vec{\epsilon_{t-l}} + \sum_{l=1}^{r} C_l I(\eta_{t-l}) \vec{\epsilon_{t-l}} + \sum_{l=1}^{s} B_l H_{t-l},$$
   where $I(\eta_{t-l})$ value 1 - return is non-positive, 0 - return is positive.
   Off-diagonal elements - spillover effects, diagonal elements - own past-time information.
2. aDCC (Asymmetric Dynamic Conditional Correlations, Engle, 2002)
   - H_t is volatility evaluated by any GARCH model
     $$H_t = D_t R_t D_t, D_t^2 = \mathrm{diag} H_t$$
   - Quasi-correlation matrix is created
     $$Q_t = \omega + \alpha \epsilon_{t-1} \epsilon_{t-1}' + \gamma \eta_{t-1} \eta_{t-1}' + \beta Q_{t-1}, \eta_t = \min[\epsilon_t, 0].$$
     If $\gamma > 0$ then correlations increase more with negative shocks.
   - Matrix is rescaled $R_t = \mathrm{diag} Q_t^{(-1/2)} Q_t \mathrm{diag} Q_t^{(-1/2)}$
3. VAR-X-mGJR/aDCC (a 2 step procedure for mean and variance, King, 1990)
   - VAR-X(p) – a vector autoregressive model with external variables - is created
     $$Y_t = C + \sum_{k=1}^{q} B Y_{t-k} + \sum_{l=1}^{L} E Y_{\text{external}} + \epsilon_t.$$
   - Residuals from VAR-X(p) are taken to further form any multivariate GARCH model.

Empirical research

Aim is to find the most risky stock, review different multivariate GARCH models and verify if adding external regressors improve model quality. For that, analysis was split into the following dimensions: mGJR-GARCH and aDCC were observed without external regressors, with change point variable and with all aforementioned variables.

	VAR-X (returns)	**mGJR-GARCH**	**aDCC**	**Dependencies to other companies**
Ryanair	Self dependence (low)	Self dependence (high)	Self dependence (low)	3
	Wizzair	EasyJet	EasyJet	
	News (Ryanair)	Wizzair	Wizzair	
		Norwegian	News (Ryanair)	
EasyJet	Self dependence (moderate)	Self dependence (high)	Self dependence (moderate)	2
	Change point	Ryanair	Wizzair	
	News (Wizzair)			
Wizzair	Self dependence (moderate)	Self dependence (moderate)	Self dependence (moderate)	1
	Ryanair	Ryanair	Ryanair	
	News (Ryanair)			
Norwegian	No self dependence	Self dependence (low)	Self dependence (high)	1
	Wizzair	Ryanair (very low)		
	Terrorist attacks			
	News (Norwegian)			

(a)

Figure: Multivariate methods comparison
Source: created by the author.

Conclusions:

- Statistically significant asymmetry was found in each company, depending on the model.
- Change points, terrorist attacks and news impact are statistically significant.
- Including external regressors highly improved model diagnostics (AIC and likelihood) and increased explainable part of volatility.
- aDCC showed slightly better model diagnostics over mGJR-GARCH.
- Risk order in the view of correlations, spillover effects and self dependency: Ryanair → EasyJet →Wizzair →Norwegian

Conclusions

- Including heavy tail assumption significantly improved model diagnostics.
- Exogenous variables increased model quality and broadened the explainable part of the volatility.
- Using models with asymmetry added value to better understand companies' different sensitivity to good or bad news, but the findings were different depending on model used, showing lack of robustness and need for further investigation.
- Evaluating the risk through it's dependency on own past time information and relation to other companie's returns or external regressors, the following risk order was found in most of the models: Ryanair → EasyJet →Wizzair →Norwegian