Comparison of Fuzzy Sets Based on the Concept of Imprecision

Jolanta Miliauskaitė[1], Diana Kalibatienė[2]
[1] Institute of Data Science and Digital Technologies, Vilnius University, jolanta.miliauskaite@mif.vu.lt
[2] Information Systems Department, Vilnius Gediminas Technical University, diana.kalibatiene@vilniustech.lt

ABSTRACT

- Since 1971 introduced different types of fuzzy sets, questions arose as to how these fuzzy sets differ and which are more suitable for a particular application domain modelling.
  - Some fuzzy sets, like type-1, type-2, etc., express fuzziness by developing the membership degree of an element.
  - Others, like intuitionistic fuzzy sets, are defined by membership and non-membership degrees of an element.
- There should be a method to implement the imprecise concept by those fuzzy sets in a reasoning engine and use it for automated inference.
  - We are dealing with the computational complexity, complexity of fuzzy rules, complexity of developing membership functions, and data complexity.
- In this research, we present an *initial study* of different types of fuzzy sets based on the concept of imprecision and their historical occurrence.
- The results allow us to supplement existing knowledge on fuzzy sets by systematizing them.

CHRONOLOGICAL ANALYSIS

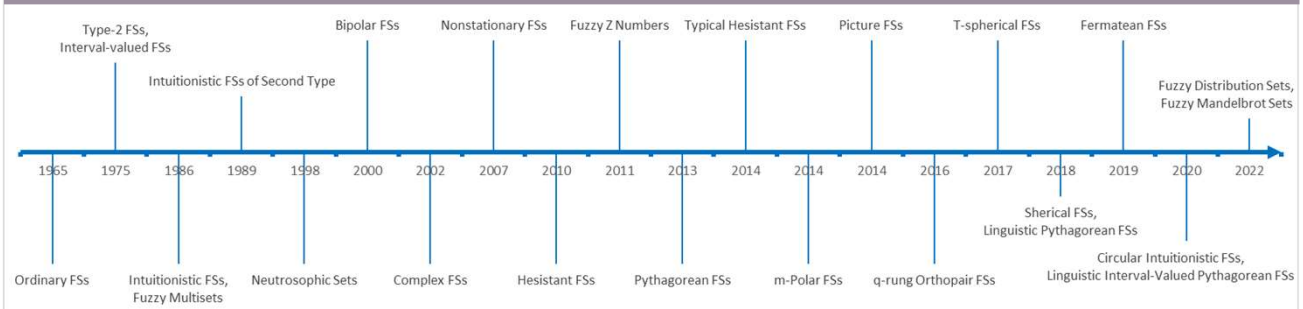
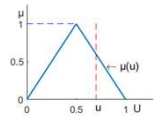


Figure 1. The chronological occurrence of fuzzy sets (CHOA)

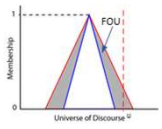
FUZZY SETS AND THEIR VISUALIZATION



Definition 1: A ***fuzzy set* (type-1 or ordinary)** [1, 4] A is a set whose elements have membership degr (MD):

$$A = \{u, \mu_A(u) | u \in U, \mu_A \colon U \to [0;\ 1]\},$$

where A in the universe of discourse U and, $\mu_A(u)$ is a membership function (MF) of u in A. The value $\mu_A(u)$ represents the grade of membership of u in A and is interpreted as the MD to which u belongs t



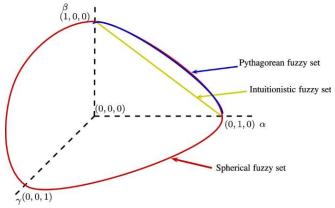
Definition 2: A ***type-2 fuzzy set*** [2] $\tilde{A}$, is characterized by a type-2 MF $\mu_{\tilde{A}}(u, x)$:

$$\tilde{A} = \{((u, x), \mu_{\tilde{A}}(u, x)) | \forall u \in U, \forall x \in J_u \in [0; 1], \mu_{\tilde{A}}(u, x) \in [0; 1]\},$$

where $u \in U$ and $x \in X$ and $x \in J_u \in [0; 1]$ – the primary membership of u [1].

Definition 2a: Uncertainty in the primary memberships of a Type-2 fuzzy set, $\tilde{A}$, consists of a bounde region that is called the ***footprint of uncertainty*** (FOU):

$$FOU(\tilde{A}) = \bigcup_{u \in U} J_u .$$



Source: [8]

Definition 3: An ***Intuitionistic fuzzy set*** A^* in U is an object of MDs described with a pair of MD and non-MD.

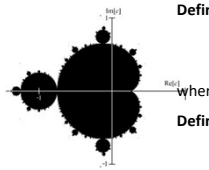
$$A^* = \{\langle u, \mu_{A^*}(u)|, (u)\rangle | u \in U, \mu_{A^*} \colon U \to [0;\ 1], \nu_{A^*} \colon E \to [0; 1]\},$$

where μ_{A^*} – define MD and ν_{A^*} – non-MD [3, 4].

Definition 4: A ***Pythagorean fuzzy set*** A^P is a set of ordered pairs over X, defined as:

$$A^P = \left\{ \left\langle \frac{\langle \mu_{A^P}(u), \nu_{A^P}(u) \rangle}{u} \right\rangle \middle| u \in U, \mu_{A^P} \colon U \to [0;\ 1], \nu_{A^P} \colon U \to [0; 1] \right\},$$

where μ_{A^P} – define MD and ν_{A^P} – non-MD [5].



Source: nLab authors (2022). Mandelbrot set. https://ncatlab.org/nlab/show/Mandelbrot%20set

Definition 5: A ***Spherical fuzzy set*** [6] is a fuzzy set defined as:

$$J = \left\{ \begin{matrix} \langle r, P_j(r), I_j(r), N_j(r) | r \in R \rangle, R \neq \varphi, P_j : R \to [0;\ 1], \\ I_j : R \to [0;\ 1], N_j : R \to [0;\ 1] \end{matrix} \right\},$$

where P_j, I_j, and N_j indicate a positive, neutral, and negative MDs of each $r \in R$, respectively.

Definition 6: A ***Fuzzy Mandelbrot set*** [7] is the fuzzy set $\widetilde{M} = \{(c, \mu(c)) | c \in \mathbb{C}, \mu \colon \mathbb{C} \to [0; 1])\}$, where

$$\mu(c) = \begin{cases} 1, & if\ |f_c^n(0)| \leq 2, \forall\ n \in \mathbb{N} \\ \left| \frac{f_c^{k-1}(0)}{f_c^k(0)} \right|, & if |f_c^n(0)| > 2\ and\ \left|f_c^{k-1}(0)\right| \leq 2, k \in \mathbb{N}' \end{cases}$$

under the iteration $f_c^n(0) = [f_c^{n-1}(0)]^2 + c$ with the initial point $z = 0 + 0_i$.

BIBLIOMETRIC ANALYSIS

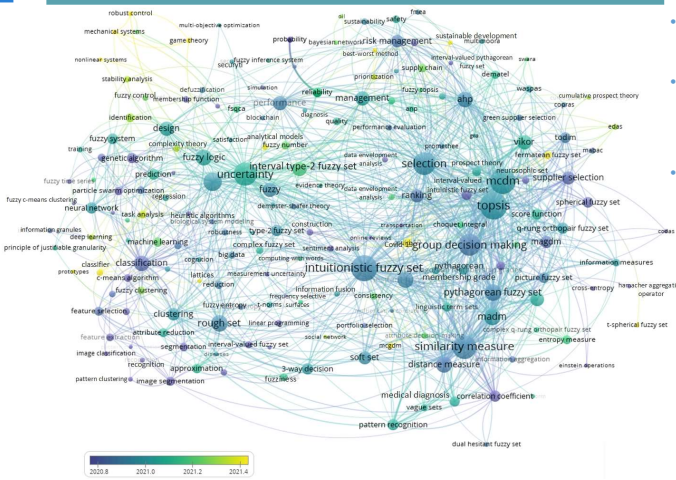


Figure 2. Keyword map of fuzzy sets (2020-2022) (BibA)

METHODS

- Chronological occurrence analysis (CHOA)

When and what types of fuzzy sets appeared in publications?

- Theoretical analysis (ThA)

What does each fuzzy set concept we find mean?

How is it defined?

- Bibliometric analysis (BibA)

What are the visible trends for the fuzzy sets appearance?

What are the main application areas of the found fuzzy sets?

CONCLUSIONS

- The chronological analysis of fuzzy sets appearance shows a trend of sets fuzzification, like spherical set → fuzzy spherical sets, Mandelbrot set → fuzzy Mandelbrot set, etc.
- The theoretical and bibliometric analysis of the found fuzzy sets and their definitions shows the growing complexity of fuzzy sets and their application in new approaches for classification or regression in MCDM, diagnosis, safety, sustainability, pattern recognition, covid-19, mechanical systems, game theory, etc.

REFERENCES

[1] Zimmermann, H. J. (2010). Fuzzy set theory. *Wiley interdisciplinary reviews: computational statistics*, 2(3), 317-332.

[2] Mendel, J. M., & John, R. B. (2002). Type-2 fuzzy sets made simple. *IEEE Transactions on fuzzy systems*, 10(2), 117-127. https://doi.org/10.1109/91.995115

[3] Atanassov, K. T. (2012). *On intuitionistic fuzzy sets theory* (Vol. 283). Springer. https://doi.org/10.1007/978-3-642-29127-2

[4] Ejegwa, P. A., Akowe, S. O., Otene, P. M., & Ikyule, J. M. (2014). An overview on intuitionistic fuzzy sets. *International Journal of Scientific & Technology Research*, 3(3), 142-145.

[5] Ejegwa, P.A. (2019). Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. *Complex & Intelligent Systems,* 5, 165–175. https://doi.org/10.1007/s40747-019-0091-6

[6] Ashraf, S., Abdullah, S., Aslam, M., Qiyas, M., & Kutbi, M. A. (2019). Spherical fuzzy sets and its representation of spherical fuzzy t-norms and t-conorms. *Journal of Intelligent & Fuzzy Systems*, 36(6), 6089-6102. https://doi.org/10.3233/JIFS-181941

[7] Mahmood, T., & Ali, Z. (2022). Fuzzy superior mandelbrot sets. *Soft Computing*, 26(18), 9011-9020. https://doi.org/10.1007/s00500-022-07254-x

[8] Akram, M., Al-Kenani, A.N. & Shabir, M. Enhancing ELECTRE I Method with Complex Spherical Fuzzy Information. *International Journal of Computational Intelligence Systems*, 14, 190 (2021). https://doi.org/10.1007/s44196-021-00038-5

Definiton 6a: The ***Mandelbrot set*** [6] is the set of complex numbers c for which the function $f_c(z) = z^2 + c$ does not diverge to infinity when iterated from $z = 0$, i.e., for which the sequence $f_c(0)$, $f_c(f_c(0))$, etc., remains bounded in absolute value.