A Projection Method for Multiple Attribute Group Decision Making with Intuitionistic Fuzzy Information

Shouzhen ZENG¹*, Tomas BALEŽENTIS², Ji CHEN³, Gangfei LUO³

¹College of Compute and Information, Zhejiang Wanli University
Ningbo 315100, China
²Lithuanian Institute of Agrarian Economics
V. Kudirkos Str. 18, LT-03105 Vilnius, Lithuania
³College of Statistics and Mathematics, Zhejiang Gongshang University
Hangzhou 310018, China

E-mail: zszzxl@163.com, tomas@laei.lt, chenji810404@163.com, purityly@163.com

Received: September 2012; accepted: April 2013

Abstract. The aim of this paper is to investigate intuitionistic fuzzy multiple attribute group decision making problems where the attribute values provided by experts are expressed in intuitionistic fuzzy numbers, and the weight information about the experts is to be determined. We present a new method to derive the weights of experts and rank the preference order of alternatives based on projection models. We first derive the weights of the decision makers according to the projection of the individual decision on the ideal decision. The expert has a large weight if his evaluation value is close to the ideal decision, and has a small weight if his evaluation value is far from the ideal decision. Then, based on the weighted projection of the alternatives on the intuitionistic fuzzy ideal solution (IFIS), we develop a straightforward and practical algorithm to rank alternatives. Furthermore, we extend the developed model and algorithm to the multiple attribute group decision making problems with interval-valued intuitionistic fuzzy information. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Key words: intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, projection method, multiple attribute group decision making.

1. Introduction

Multiple attribute group decision making problems are those of major importance in diverse fields such as engineering, economics, and management. The current socio-economic environment is becoming more and more complex, which makes it almost impossible for a single decision maker to consider all the aspects of a problem (Liou and Tzeng, 2012). Generally, several decision makers are involved in the decision making. In the process of decision making, the decision information about alternatives is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and
the vagueness of inherent subjective nature of human thinking. The intuitionistic fuzzy set (IFS) (Atanassov, 1986, 1999), characterized by a membership function and a non-membership function, is more suitable for dealing with fuzziness and uncertainty than the ordinary fuzzy set developed by Zadeh (1965) whose basic component is only a membership function. Gau and Buehrer (1993) gave the notion of vague set, which is another generalization of fuzzy sets. Bustince and Burillo (1996), nevertheless, showed that it is an equivalent of the IFS. Consider that, sometimes, it is not approximate to assume that the membership degrees for certain elements of an IFS are exactly defined, but a value range can be given. In such cases, Atanassov and Gargov (1989) introduced the notion of interval-valued intuitionistic fuzzy set (IVIFS), which is characterized by a membership function and a nonmembership function, whose values are intervals rather than exact numbers.

The IFS and IVIFS are highly useful in dealing with fuzziness and uncertainty, and thus, recently many researchers have applied them to the complex decision making problems. Some methods have been developed to solve the single person multiple attribute decision making problems with the IFS and IVIFS information (Grzegorzewski, 2004; Liu and Wang, 2007; Lin et al., 2007; Gong et al., 2009; Li, 2004, 2008, 2011; Wei, 2008, 2010a; Xu, 2007a, 2007b; Xu and Yager, 2008; Zhao et al., 2010; Lakshmana Gomathi Nayagam and Geetha, 2011; Wang et al., 2011; Ye, 2009, 2010). On multi-person multi-attribute decision making, alternatively called multi-attribute group decision making (MAGDM, for short) problems, Mitchell (2004) defined an intuitionistic OWA operator which aggregates a set of intuitionistic fuzzy sets and described a simple application of the new intuitionistic OWA operator in multiple-expert multi-criteria decision making. Atanassov et al. (2005) provided a tool to solve the multi-person multi-criteria decision making problems, in which the attribute weights are given as exact numerical values and the attribute values are expressed in intuitionistic fuzzy numbers. Xu and Yager (2006) developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave an application of the IFHG operator to MAGDM with intuitionistic fuzzy information. Xu (2007c) investigated the group decision making problems in which all the information provided by the decision makers is expressed as intuitionistic fuzzy decision matrices where each of the elements is characterized by intuitionistic fuzzy number, and the information about attribute weights is partially known. Xu (2007d) developed an approach to group decision making based on intuitionistic preference relations. Xu and Yager (2009) developed a new similarity measure and apply the developed similarity measure to the consensus analysis in group decision making based on intuitionistic fuzzy preference relations. Boran et al. (2009) combined TOPSIS method with IFS to select appropriate supplier in group decision making environment. Li et al. (2009) developed a new methodology based on some fractional programming models and the ranking method for solving MAGDM problems using intuitionistic fuzzy sets (IFSs). Li et al. (2010) developed a linear programming methodology for solving MAGDM problems using IFSs. Yue et al. (2009) introduced an approach for aggregating multiple attribute values characterized by precise numerical values into an intuitionistic fuzzy number and gave an
A Projection Method for Multiple Attribute Group Decision Making

Wei (2010b) developed the induced intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator, and presented its application to MAGDM problems. Xu (2010a) utilized distance measures to solve MAGDM problems with interval-valued intuitionistic fuzzy information. Xu (2010b) investigated the MAGDM with intuitionistic fuzzy information and information about attribute weights is completely known or completely unknown, and proposed a deviation-based approach to solve the problems. Xu and Wang (2012) presented the induced generalized intuitionistic fuzzy ordered weighted averaging (IGIFOWA) operator and applied it to MAGDM problems concerning with searching the best global supplier. Zeng and Su (2011) and Zeng (in press) presented an intuitionistic fuzzy ordered weighted distance (IFOWD) operator, and applied it to group decision making. In all these literature, the weights of the decision makers (or experts) are determined beforehand. At present, many methods have been proposed to determine the weights of experts in intuitionistic fuzzy multiple attribute group decision making problems. Tan (2011) determined the decision makers’ weights by the means of the Choquet integral. Xu and Cai (2010) developed some nonlinear optimization models to get the decision makers’ weights. Yue (2011a) developed a new approach for measuring the decision makers’ weights in group decision making setting based on distance measure, in which the decision information is expressed in interval-valued intuitionistic fuzzy numbers. Yue (2011b) presented an approach for group decision making based on determining weights of experts using TOPSIS method. Recently, Yue (2012a) introduced an approach for group decision making based on determining the weights of experts by using projection method. Yue (2012b) also developed a projection method for determining weights of DMs with interval numbers. Wang et al. (2009) investigated the group decision making problems in featuring incompletely known weights of the attributes and decision makers, which may be constructed by employing intuitionistic fuzzy numbers (IFNs). In all these existing approaches, the weights of the experts are the same for all the attributes. However, if the weights of experts for all the attributes are the same, the evaluating result would be unreasonable. Hence, the different weights of the decision makers should be assigned to different attributes in the group decision making problem, as different experts have their own knowledge and experience in reality and they are actually experts in some of the attributes and not in other attributes. Inspired by this idea, in this paper, we propose a projection method to derive experts’ weights in IFSs and IVIFSs MAGDM through aggregating the individual decision matrices into a collective decision matrix. Especially, the expert whose evaluation value is close to the ideal decision has a large weight, while the expert whose evaluation value is far from the ideal decision would have a small weight. Further, the preference order of alternatives can be ranked in accordance with the projections of alternatives onto the ideal solution.

The rest of the paper is organized as follows. In Section 2, we give some basic concepts. In Section 3, we propose a straightforward and practical method to derive the weights of experts and rank the preference order of alternatives based on projection models. In Section 4, we extend the developed models and procedures to handle the MAGDM problems with interval-valued intuitionistic fuzzy information. In Section 5, we illustrate our proposed algorithmic method with an example. The final section concludes.
2. Preliminaries

Atanassov introduced the concept of intuitionistic fuzzy set (IFS), which was defined as follows:

Let a set $X$ be fixed, an intuitionistic fuzzy set $A$ in $X$ is an object having the following form:

$$A = \{ [x, \mu_A(x), v_A(x)] | x \in X \}$$  \hspace{1cm} (1)

where the functions $\mu_A(x) : X \rightarrow [0, 1]$ and $v_A(x) : X \rightarrow [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x \in X$, such that $0 \leq \mu_A(x) + v_A(x) \leq 1$ for all $x \in X$. In addition $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of indeterminacy of $x$ to $A$, or called the degree of hesitancy of $x$ to $A$. Especially, if $\pi_A(x) = 0$, for all $x \in X$, then the IFS $A$ is reduced to a fuzzy set.

For convenience, the $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$ is called the intuitionistic fuzzy number (IFN) (Xu, 2007a; Xu and Yager, 2009), where

$$\mu_\alpha \in [0, 1], \hspace{0.5cm} v_\alpha \in [0, 1], \hspace{0.5cm} \mu_\alpha + v_\alpha \leq 1, \hspace{0.5cm} \pi_\alpha = 1 - \mu_\alpha - v_\alpha$$  \hspace{1cm} (2)

and denote the module of $\alpha$ as:

$$|\alpha| = \sqrt{\mu_\alpha^2 + v_\alpha^2 + \pi_\alpha^2}.$$  \hspace{1cm} (3)

**Definition 1.** (See Xu and Hu, 2010.) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, $A$ be an IFS in $X$, then

$$|A| = \sqrt{\sum_{i=1}^{n} |\alpha_i|^2}$$  \hspace{1cm} (4)

is called the module of $A$, where $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i})$ is the $i$-th IFN of $A$.

In many situations, the weight of the element $x_j \in X$ should be taken into account, for example, in multiple attribute decision making, the considered attributes usually have different importance, and thus need to be assigned with different weights. Xu and Hu (2010) defined the weighted module as follows.

**Definition 2.** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, $A$ be an IFS in $X$, then

$$|A|_w = \sqrt{\sum_{i=1}^{n} (w_i |\alpha_i|^2)}$$  \hspace{1cm} (5)

is called the weighted module of $A$, where $\alpha_i = (\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i})$ is the $i$-th IFN of $A$, and $w = (w_1, w_2, \ldots, w_n)$ is the weighting vector of $x_j$ ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1], \sum_{j=1}^{n} w_j = 1$. 
A Projection Method for Multiple Attribute Group Decision Making

DEFINITION 3. (See Xu and Hu, 2010.) Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a finite universe of discourse, \( A \) and \( B \) be two IFSs in \( X \), then

\[
\text{Pr}_B A = \frac{1}{|B|} \sum_{i=1}^{n} (\mu_{\alpha_i} \mu_{\beta_i} + v_{\alpha_i} v_{\beta_i} + \pi_{\alpha_i} \pi_{\beta_i})
\]

(6)

is called the projection of \( A \) on \( B \), where \( \alpha_i = (\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i}) \) and \( \beta_i = (\mu_{\beta_i}, v_{\beta_i}, \pi_{\beta_i}) \) are the \( i \)-th IFNs of \( A \) and \( B \), respectively. Obviously, the greater the value \( \text{Pr}_B A \), the more the degree of the \( A \) approaching to the \( B \). Especially, if \( n = 1 \), then we get the the projection of IFN \( \alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1}, \pi_{\alpha_1}) \) on \( \beta_1 = (\mu_{\beta_1}, v_{\beta_1}, \pi_{\beta_1}) \) as:

\[
\text{Pr}_\beta \alpha_1 = \frac{1}{|\beta_1|} (\mu_{\alpha_1} \mu_{\beta_1} + v_{\alpha_1} v_{\beta_1} + \pi_{\alpha_1} \pi_{\beta_1}).
\]

(7)

DEFINITION 4. (See Xu and Hu, 2010.) Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a finite universe of discourse, \( A \) and \( B \) be two IFSs in \( X \), then

\[
\text{Pr}_B A = \frac{1}{|B|} \sum_{i=1}^{n} w_i^2 (\mu_{\alpha_i} \mu_{\beta_i} + v_{\alpha_i} v_{\beta_i} + \pi_{\alpha_i} \pi_{\beta_i})
\]

(8)

is called the weighted projection of \( A \) on \( B \), where \( \alpha_i = (\mu_{\alpha_i}, v_{\alpha_i}, \pi_{\alpha_i}) \) and \( \beta_i = (\mu_{\beta_i}, v_{\beta_i}, \pi_{\beta_i}) \) are the \( i \)-th IFNs of \( A \) and \( B \), respectively. \( w = (w_1, w_2, \ldots, w_n) \) is the weighting vector of \( x_j \) (\( j = 1, 2, \ldots, n \)) with \( w_j \in [0, 1] \), \( \sum_{j=1}^{n} w_j = 1 \).

3. Multiple Attribute Group Decision Making Based on Intuitionistic Fuzzy Sets

In the multiple attribute group decision making, decision makers first determine the evaluation values of the alternatives with respect to attributes. With these evaluation values, decision makers select the best alternative or rank the alternatives. During the decision process, several experts are involved in order to get a reasonable result. In the existing group decision making problems, the weights of the decision makers or experts are often determined beforehand. Usually, they have the same weight for all the attributes. But in some real-life situations, some decision makers are familiar with some of the attributes, but not others due to any activities requiring human expertise and knowledge, which are inevitably imprecise or not totally reliable. When evaluating attributes they are not good at, experts may provide unreasonable values. The result would be unreasonable if the weights of experts for all the attributes are the same. On the other hand, if we assign different expert weights to different attributes, the additional amount of work would be too large. In this section, instead of assigning the weights of experts, we develop a projection model to derive weights of the experts from the evaluation values. The expert whose evaluation value is close to the ideal decision should be assigned a large weight, while the expert whose evaluation value is far from the ideal decision would have a small weight.
For a group decision making problem, let \( A = \{A_1, A_2, \ldots, A_m\} \) be a finite set of alternatives, \( D = \{D_1, D_2, \ldots, D_t\} \) be the set of decision makers (or experts), \( G = \{G_1, G_2, \ldots, G_n\} \) be the set of attributes, and \( w = (w_1, w_2, \ldots, w_n) \) be the weighting vector of the attributes, here \( w_j \in [0, 1], \sum_{j=1}^{n} w_j = 1 \). An intuitionistic fuzzy decision matrix \( R^{(k)} = (r^{(k)}_{ij})_{m \times n} = (t^{(k)}_{ij}, f^{(k)}_{ij}, \pi^{(k)}_{ij})_{m \times n} \), whose elements \((t^{(k)}_{ij}, f^{(k)}_{ij}, \pi^{(k)}_{ij})\) are IFNs, is provided by the decision maker \( D_k \) for the alternative \( A_i \) with respect to the attribute \( G_j \). Here, given by the decision maker \( D_k \), \( t^{(k)}_{ij} \) indicates the degree that the alternative \( A_i \) should satisfy the attribute \( G_j \), whereas there are generally benefit attributes (the bigger the attribute values the better) and cost attributes (the smaller the attribute values the better) in MAGDM. In such cases, we may transform the attribute values of cost type into the attribute values of benefit type, then \( R^{(k)} = (t^{(k)}_{ij})_{m \times n} \) can be transformed into the intuitionistic fuzzy decision matrices \( D^{(k)} = (d^{(k)}_{ij})_{m \times n} \), where

\[
\begin{align*}
    d^{(k)}_{ij} &= (\mu^{(k)}_{ij}, v^{(k)}_{ij}, \pi^{(k)}_{ij}) \\
    &= \begin{cases} 
        (t^{(k)}_{ij}, f^{(k)}_{ij}, \pi^{(k)}_{ij}), & \text{for benefit attribute } G_j, \\
        (f^{(k)}_{ij}, t^{(k)}_{ij}, \pi^{(k)}_{ij}), & \text{for cost attribute } G_j,
    \end{cases}
\end{align*}
\]

Suppose that the evaluation values for the alternative \( A_i \) with respect to the attribute \( G_j \) are \( d^{(1)}_{ij} = (\mu^{(1)}_{ij}, v^{(1)}_{ij}, \pi^{(1)}_{ij}), d^{(2)}_{ij} = (\mu^{(2)}_{ij}, v^{(2)}_{ij}, \pi^{(2)}_{ij}), \ldots, d^{(t)}_{ij} = (\mu^{(t)}_{ij}, v^{(t)}_{ij}, \pi^{(t)}_{ij}) \) provided by \( t \) experts. We define the mean of these evaluation values as \( d^*_{ij} = (\mu^*_{ij}, v^*_{ij}, \pi^*_{ij}) \), where

\[
\begin{align*}
    \mu^*_{ij} &= \frac{1}{t} \sum_{k=1}^{t} \mu^{(k)}_{ij}, \\
    v^*_{ij} &= \frac{1}{t} \sum_{k=1}^{t} v^{(k)}_{ij}, \\
    \pi^*_{ij} &= \frac{1}{t} \sum_{k=1}^{t} \pi^{(k)}_{ij}.
\end{align*}
\]

Inspired by compromise elements in the literature (Yue, 2011a, 2011b, 2012), the mean value \( d^*_{ij} = (\mu^*_{ij}, v^*_{ij}, \pi^*_{ij}) \) can be defined as the ideal decision of all these evaluation values \( d^{(k)}_{ij} \) \( (k = 1, 2, \ldots, t) \). In this sense, the more the degree that \( d^{(k)}_{ij} \) is closer to the \( d^*_{ij} \), the better the decision \( d^{(k)}_{ij} \). Therefore, we can calculate the projection of each evaluation value on the ideal decision \( d^*_{ij} \) by (7):

\[
\Pr(d^*_{ij} \parallel d^{(k)}_{ij}) = \frac{1}{|d^*_{ij}|} (\mu^{(k)}_{ij} \mu^*_{ij} + v^{(k)}_{ij} v^*_{ij} + \pi^{(k)}_{ij} \pi^*_{ij}).
\]
Then the weight for \( d_{ij}^{(k)} \) can be defined as:

\[
\begin{align*}
    w_{ij}^{(k)} &= \frac{\Pr_j d_{ij}^{(k)}}{\sum_{k=1}^t \Pr_j d_{ij}^{(k)}}, \\
    & \quad k = 1, 2, \ldots, t, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.
\end{align*}
\]  \hfill (13)

The weight of each expert determined by this model has the following desirable characteristic: the closer an evaluation value is to the mean value, the larger the weight is. In this algorithm, the experts’ weights are different for different attributes. This can avoid the unreasonable evaluation value induced by decision makers’ limited knowledge or experience.

When the weight values for the experts are determined, the evaluating values provided by different experts can be aggregated by using the weighted averaging operator:

\[
\begin{align*}
    d_{ij} &= w_{ij}^{(1)} d_{ij}^{(1)} + w_{ij}^{(2)} d_{ij}^{(2)} + \cdots + w_{ij}^{(t)} d_{ij}^{(t)}.
\end{align*}
\]  \hfill (14)

Therefore, we can obtain the collective decision matrix \( D = (d_{ij})_{m \times n} \).

In the following, we will propose a procedure for MAGMD problems with intuitionistic fuzzy information by application of projection models. The procedure involves the following steps:

**Algorithm 1**

**Step 1.** The decision makers evaluate the alternatives with respect to the attributes to form the intuitionistic fuzzy decision matrices. Determine the expert weights for each evaluation value by (12) and (13), and aggregate the different experts’ evaluations into a collective one by (14).

**Step 2.** Calculate the intuitionistic fuzzy ideal solution (IFIS): \( A^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_m^*) \), where \( \alpha_j^* = (\mu_j^*, v_j^*, \pi_j^*) \) is IFN, and

\[
\begin{align*}
    \mu_j^* &= \max_i \{\mu_{ij}\}, \\
    v_j^* &= \min_i \{v_{ij}\}, \\
    \pi_j^* &= 1 - \mu_j^* - v_j^*.
\end{align*}
\]  \hfill (15)

**Step 3.** Utilize (8) to calculate the weighted projection of the alternative \( A_i \) (\( i = 1, 2, \ldots, m \)) on the IFIS \( A^* \), that is

\[
\begin{align*}
    \Pr_j A^* A_i &= \frac{1}{|A^*|_w} \sum_{j=1}^n w_j^2 (\mu_{ij} \mu_j^* + v_{ij} v_j^* + \pi_{ij} \pi_j^*).
\end{align*}
\]  \hfill (16)

**Step 4.** Rank all the alternatives \( A_i \) (\( i = 1, 2, \ldots, m \)) in accordance with the projection \( \Pr_j A^* A_i \). Obviously, the larger the projection \( \Pr_j A^* A_i \), the closer the alternative \( A_i \) is to the IFIS \( A^* \), and the better the alternative, \( A_i \). The above method employs only the projection models and the weighted averaging operator to aggregate evaluation information, therefore it is very simple and convenient to use in practical applications.
In what follows, we shall extend the developed methods to the MAGDM with the interval-valued intuitionistic fuzzy information.

4. Multiple Attribute Group Decision Making Based on Interval-Valued Intuitionistic Fuzzy Sets

Interval-valued intuitionistic fuzzy set (IVIFS) was first introduced by Atanassov and Gargov (1989). It is characterized by an interval-valued membership degree and an interval-valued non-membership degree. Let a set \( X \) be fixed, an IVIFS in \( X \) is an object of the following form

\[
\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) \mid x \in X\}
\]

where \( \mu_{\tilde{A}}(x) = [\mu_{L\tilde{A}}(x), \mu_{U\tilde{A}}(x)] \subseteq [0, 1] \) and \( \nu_{\tilde{A}}(x) = [\nu_{L\tilde{A}}(x), \nu_{U\tilde{A}}(x)] \subseteq [0, 1] \) are intervals, \( \mu_{L\tilde{A}}(x) = \inf \mu_{\tilde{A}}(x), \mu_{U\tilde{A}}(x) = \sup \mu_{\tilde{A}}(x), \nu_{L\tilde{A}}(x) = \inf \nu_{\tilde{A}}(x), \nu_{U\tilde{A}}(x) = \sup \nu_{\tilde{A}}(x), \) and

\[
\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1, \quad \text{for all} \ x \in X.
\]

Let \( \pi_{\tilde{A}}(x) = [\pi_{L\tilde{A}}(x), \pi_{U\tilde{A}}(x)] \), where

\[
\pi_{L\tilde{A}}(x) = 1 - \mu_{U\tilde{A}}(x) - \nu_{U\tilde{A}}(x), \quad \pi_{U\tilde{A}}(x) = 1 - \mu_{L\tilde{A}}(x) - \nu_{L\tilde{A}}(x), \quad \text{for all} \ x \in X.
\]

Especially, if \( \mu_{\tilde{A}}(x) = \mu_{L\tilde{A}}(x) = \tilde{\mu}_{L\tilde{A}}(x) \) and \( \nu_{\tilde{A}}(x) = \nu_{L\tilde{A}}(x) = \tilde{\nu}_{L\tilde{A}}(x) \), then \( \tilde{A} \) is reduced to an IFS.

The pair \( (\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x), \pi_{\tilde{A}}(x)) \) is called an interval-valued intuitionistic fuzzy number (IVIFN) (Xu and Yager, 2009). For convenience, we denote an IVIFN by \( \tilde{a} = (\tilde{\mu}_a, \tilde{\nu}_a, \tilde{\pi}_a) \), where

\[
\tilde{\mu}_a = [\mu_{L\tilde{a}}, \mu_{U\tilde{a}}] \subseteq [0, 1], \quad \tilde{\nu}_a = [\nu_{L\tilde{a}}, \nu_{U\tilde{a}}] \subseteq [0, 1], \quad \tilde{\mu}_{L\tilde{a}} + \tilde{\nu}_{U\tilde{a}} \leq 1,
\]

\[
\tilde{\pi}_a = [\pi_{L\tilde{a}}, \pi_{U\tilde{a}}] = [1 - \tilde{\mu}_{U\tilde{a}} - \tilde{\nu}_{U\tilde{a}}, 1 - \tilde{\mu}_{L\tilde{a}} - \tilde{\nu}_{L\tilde{a}}].
\]

Definition 5. (See Xu and Hu, 2010.) Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a finite universe of discourse, \( \tilde{A} \) be an IVIFS in \( X \), then

\[
|\tilde{A}| = \sqrt{\sum_{i=1}^{n} |\tilde{a}_i|^2}
\]
is called the module of $\tilde{A}$, where $\tilde{a}_i = (\tilde{\mu}_{a_i}, \tilde{\nu}_{a_i}, \tilde{\pi}_{a_i})$ is the $i$-th IVIFN of $\tilde{A}$, and $|\tilde{a}_i|$ is the module of $\tilde{a}_i$, which can be denoted as follows:

$$|\tilde{a}_i| = \sqrt{(\tilde{\mu}_{a_i} L)^2 + (\tilde{\mu}_{a_i} U)^2 + (\tilde{\nu}_{a_i} L)^2 + (\tilde{\nu}_{a_i} U)^2 + (\tilde{\pi}_{a_i} L)^2 + (\tilde{\pi}_{a_i} U)^2}.$$  (22)

**Definition 6.** (See Xu and Hu, 2010.) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, $\tilde{A}$ be an IVIFS in $X$, then

$$|\tilde{A}|_w = \sqrt{\sum_{i=1}^{n} (w_i |\tilde{a}_i|)^2}.$$  (23)

is called the weighted module of $\tilde{A}$, where $\tilde{a}_i = (\tilde{\mu}_{a_i}, \tilde{\nu}_{a_i}, \tilde{\pi}_{a_i})$ is the $i$-th IVIFN of $\tilde{A}$, and $w = (w_1, w_2, \ldots, w_n)$ is the weighting vector of $x_j$ ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$.

**Definition 7.** (See Xu and Hu, 2010.) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, $\tilde{A}$ and $\tilde{B}$ be two IVIFSs in $X$, then

$$\Pr_{\tilde{B}} \tilde{A} = \frac{\sum_{i=1}^{n} (\tilde{\mu}_{a_i} L + \tilde{\mu}_{a_i} U + \tilde{\nu}_{a_i} L + \tilde{\nu}_{a_i} U + \tilde{\pi}_{a_i} L + \tilde{\pi}_{a_i} U)}{|\tilde{B}|}.$$  (24)

is called the projection of $\tilde{A}$ on $\tilde{B}$, where $\tilde{a}_i = (\tilde{\mu}_{a_i}, \tilde{\nu}_{a_i}, \tilde{\pi}_{a_i})$ and $\tilde{b}_i = (\tilde{\mu}_{b_i}, \tilde{\nu}_{b_i}, \tilde{\pi}_{b_i})$ are the $i$-th IVIFNs of $\tilde{A}$ and $\tilde{B}$, respectively. Obviously, the greater the value $\Pr_{\tilde{B}} \tilde{A}$, the more the degree of the $\tilde{A}$ approaching to the $\tilde{B}$. Especially, if $n = 1$, then we get the the projection of IVIFN $\tilde{a}_1 = (\tilde{\mu}_{a_1}, \tilde{\nu}_{a_1}, \tilde{\pi}_{a_1})$ on $\tilde{b}_1 = (\tilde{\mu}_{b_1}, \tilde{\nu}_{b_1}, \tilde{\pi}_{b_1})$ as:

$$\Pr_{\tilde{b}_1} \tilde{a}_1 = \frac{1}{|\tilde{b}_1|} (\tilde{\mu}_{a_1} L + \tilde{\mu}_{a_1} U + \tilde{\nu}_{a_1} L + \tilde{\nu}_{a_1} U + \tilde{\pi}_{a_1} L + \tilde{\pi}_{a_1} U).$$  (25)

**Definition 8.** (See Xu and Hu, 2010.) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, $\tilde{A}$ and $\tilde{B}$ be two IVIFSs in $X$, then

$$\Pr_{\tilde{B}} \tilde{A} = \frac{\sum_{i=1}^{n} w_i (\tilde{\mu}_{a_i} L + \tilde{\mu}_{a_i} U + \tilde{\nu}_{a_i} L + \tilde{\nu}_{a_i} U + \tilde{\pi}_{a_i} L + \tilde{\pi}_{a_i} U)}{|\tilde{B}|_w}.$$  (26)

is called the weighted projection of $\tilde{A}$ on $\tilde{B}$, where $\tilde{a}_i = (\tilde{\mu}_{a_i}, \tilde{\nu}_{a_i}, \tilde{\pi}_{a_i})$ and $\tilde{b}_i = (\tilde{\mu}_{b_i}, \tilde{\nu}_{b_i}, \tilde{\pi}_{b_i})$ are the $i$-th IVIFNs of $\tilde{A}$ and $\tilde{B}$, respectively. $w = (w_1, w_2, \ldots, w_n)$ is the weighting vector of $x_j$ ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$.

We consider the same decision making problem as that in the Section 3. But now the evaluation value of the alternative $A_i$ with respect to the attribute $G_j$ is represented by the IVIFNs. Let $\tilde{R}^{(k)} = (\tilde{r}^{(k)}_{ij})_{m \times n} = (\tilde{r}^{(k)}_{ij}, f^{(k)}_{ij}, \tilde{r}^{(k)}_{ij})_{m \times n}$ be an interval-valued intuitionistic
fuzzy decision matrix. \((t_{ij}^{(k)}, \tilde{f}_{ij}^{(k)}, \tilde{\pi}_{ij}^{(k)})\) is the corresponding IVIFN provided by the decision maker \(D_k\) for the alternative \(A_i\) with respect to the attribute \(G_j\). Here, \(t_{ij}^{(k)}\) indicates the degree that the alternative \(A_i\) should satisfy the attribute \(G_j\), \(\tilde{f}_{ij}^{(k)}\) indicates the degree that the alternative \(A_i\) should not satisfy the attribute \(G_j\) and \(\tilde{\pi}_{ij}^{(k)}\) indicates the degree that the alternative \(A_i\) is indeterminacy to the attribute \(G_j\), for convenience of calculation, let \(t_{ij}^{(k)} = [\tilde{t}_{ij}^{L(k)}, \tilde{t}_{ij}^{U(k)}], \tilde{f}_{ij}^{(k)} = [\tilde{f}_{ij}^{L(k)}, \tilde{f}_{ij}^{U(k)}], \tilde{\pi}_{ij}^{(k)} = [\tilde{\pi}_{ij}^{L(k)}, \tilde{\pi}_{ij}^{U(k)}]\) and

\[
\begin{align*}
[\tilde{t}_{ij}^{L(k)}, \tilde{t}_{ij}^{U(k)}] \subset [0, 1], & \quad [\tilde{f}_{ij}^{L(k)}, \tilde{f}_{ij}^{U(k)}] \subset [0, 1], & \quad \tilde{t}_{ij}^{(k)} + \tilde{f}_{ij}^{(k)} \leq 1, \\
\tilde{\pi}_{ij}^{L(k)} = 1 - \tilde{t}_{ij}^{U(k)} - \tilde{f}_{ij}^{U(k)}, & \quad \tilde{\pi}_{ij}^{U(k)} = 1 - \tilde{t}_{ij}^{L(k)} - \tilde{f}_{ij}^{L(k)}, & \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n.
\end{align*}
\]

In the cases where the attributes are of benefit and cost types, we normalize \(\tilde{R}^{(k)} = (\tilde{a}_{ij}^{(k)})_{m \times n}\) into the interval intuitionistic fuzzy decision matrices \(\tilde{D}^{(k)} = (\tilde{d}_{ij}^{(k)})_{m \times n}\), where

\[
\tilde{d}_{ij}^{(k)} = (\tilde{\mu}_{ij}^{(k)}, \tilde{\nu}_{ij}^{(k)}, \tilde{\pi}_{ij}^{(k)}) = \left([\tilde{\mu}_{ij}^{L(k)}, \tilde{\nu}_{ij}^{L(k)}], [\tilde{\nu}_{ij}^{U(k)}, \tilde{\nu}_{ij}^{U(k)}], [\tilde{\pi}_{ij}^{L(k)}, \tilde{\pi}_{ij}^{U(k)}]\right)
\]

\[
= \begin{cases} 
(\tilde{t}_{ij}^{L(k)}, \tilde{t}_{ij}^{U(k)}, \tilde{\pi}_{ij}^{(k)}), & \text{for benefit attribute } G_j, \\
(\tilde{f}_{ij}^{L(k)}, \tilde{f}_{ij}^{U(k)}, \tilde{\pi}_{ij}^{(k)}), & \text{for cost attribute } G_j,
\end{cases} \quad j = 1, 2, \ldots, n. \tag{28}
\]

Suppose that the evaluation values for the alternative \(A_i\) with respect to the attribute \(G_j\) are \(\tilde{d}_{ij}^{(1)} = (\tilde{\mu}_{ij}^{(1)}, \tilde{\nu}_{ij}^{(1)}, \tilde{\pi}_{ij}^{(1)}), \tilde{d}_{ij}^{(2)} = (\tilde{\mu}_{ij}^{(2)}, \tilde{\nu}_{ij}^{(2)}, \tilde{\pi}_{ij}^{(2)}), \ldots, \tilde{d}_{ij}^{(t)} = (\tilde{\mu}_{ij}^{(t)}, \tilde{\nu}_{ij}^{(t)}, \tilde{\pi}_{ij}^{(t)})\) provided by \(t\) experts. We define the mean of these evaluation values as \(\tilde{d}_{ij}^* = (\tilde{\mu}_{ij}^*, \tilde{\nu}_{ij}^*, \tilde{\pi}_{ij}^*) = ([\tilde{\mu}_{ij}^L, \tilde{\mu}_{ij}^U], [\tilde{\nu}_{ij}^L, \tilde{\nu}_{ij}^U], [\tilde{\pi}_{ij}^L, \tilde{\pi}_{ij}^U])\), where

\[
\begin{align*}
\tilde{\mu}_{ij}^L &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\mu}_{ij}^{L(k)}, & \quad \tilde{\mu}_{ij}^U &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\mu}_{ij}^{U(k)}, & \quad \tilde{\nu}_{ij}^L &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\nu}_{ij}^{L(k)}, & \quad \tilde{\nu}_{ij}^U &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\nu}_{ij}^{U(k)}, \\
\tilde{\nu}_{ij}^* &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\nu}_{ij}^{U(k)}, & \quad \tilde{\pi}_{ij}^L &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\pi}_{ij}^{L(k)}, & \quad \tilde{\pi}_{ij}^U &= \frac{1}{t} \sum_{k=1}^{t} \tilde{\pi}_{ij}^{U(k)}.
\end{align*}
\]

The projection of each evaluation value \(\tilde{d}_{ij}^{(k)}\) on the mean value \(\tilde{d}_{ij}^*\) is defined as follows:

\[
\Pr \tilde{f}_{ij}^{*}, \tilde{d}_{ij}^{(k)}
\]

\[
= \frac{(\tilde{\mu}_{ij}^{L(k)} \tilde{\nu}_{ij}^L + \tilde{\mu}_{ij}^{U(k)} \tilde{\nu}_{ij}^U + \tilde{\nu}_{ij}^{L(k)} \tilde{\nu}_{ij}^L + \tilde{\nu}_{ij}^{U(k)} \tilde{\nu}_{ij}^U + \tilde{\pi}_{ij}^{L(k)} \tilde{\nu}_{ij}^L + \tilde{\pi}_{ij}^{U(k)} \tilde{\nu}_{ij}^U)}{|\tilde{d}_{ij}^*|}.
\]

\[
\tag{30}
\]
Therefore, the weight for $d_{ij}^{(k)}$ can be defined as follows:

$$
u_{ij}^{(k)} = \frac{\Pr d_{ij}^{(k)}}{\sum_{k=1}^{t} \Pr d_{ij}^{(k)}}, \quad k = 1, 2, \ldots, t, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \quad (31)$$

After obtaining these weights, we can aggregate the evaluation values provided by different experts through

$$\tilde{d}_{ij} = \nu_{ij}^{(1)} \tilde{d}_{ij}^{(1)} + \nu_{ij}^{(2)} \tilde{d}_{ij}^{(2)} + \cdots + \nu_{ij}^{(t)} \tilde{d}_{ij}^{(t)}, \quad (32)$$

where $\tilde{d}_{ij} = (\tilde{\mu}_{ij}, \tilde{\nu}_{ij}, \tilde{\pi}_{ij}) = ([\tilde{\mu}_{ij}^L, \tilde{\mu}_{ij}^U], [\tilde{\nu}_{ij}^L, \tilde{\nu}_{ij}^U], [\tilde{\pi}_{ij}^L, \tilde{\pi}_{ij}^U])$, thus, we can obtain the desired collective decision matrix $\tilde{D} = (\tilde{d}_{ij})_{m \times n}$.

Similar to Section 3, a procedure for solving the above problems with interval-valued intuitionistic fuzzy information by application of projection method can be described as follows:

**Algorithm 2**

**Step 1.** The decision makers evaluate the alternatives with respect to the attributes to form the interval-valued intuitionistic fuzzy decision matrices. Determine the expert weights for each evaluation value by (30) and (31), and aggregate the different experts’ evaluations into a collective one by (32).

**Step 2.** Calculate the uncertain intuitionistic fuzzy ideal solution (UIFIS): $\tilde{A}^* = (\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_n^*)$, where $\tilde{a}_j^* = (\tilde{\mu}_j^*, \tilde{\nu}_j^*, \tilde{\pi}_j^*)$ is IVIFN, and

$$\tilde{\mu}_j^* = [(\mu_j^{L^*}, \mu_j^{U^*})] = [\max_t \{\tilde{\mu}_{ij}^t\}, \max_t \{\tilde{\mu}_{ij}^U\}],$$

$$\tilde{\nu}_j^* = [(\nu_j^{L^*}, \nu_j^{U^*})] = [\min_t \{\tilde{\nu}_{ij}^L\}, \min_t \{\tilde{\nu}_{ij}^U\}],$$

$$\tilde{\pi}_j^* = [(\pi_j^{L^*}, \pi_j^{U^*})], \quad \tilde{\pi}_j^{L^*} = 1 - \tilde{\mu}_j^{L^*} - \tilde{\nu}_j^{U^*}, \quad \tilde{\pi}_j^{U^*} = 1 - \tilde{\mu}_j^{U^*} - \tilde{\nu}_j^{L^*}, \quad (33)$$

$$i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n.$$

**Step 3.** Calculate the weighted projection of the alternative $A_i$ $(i = 1, 2, \ldots, m)$ on the UIFIS $\tilde{A}^*$ by (26):

$$\Pr j_{\tilde{A}^*} A_i = \frac{\sum_{j=1}^{n} w_j^2 (\tilde{\mu}_{ij} L^* + \tilde{\nu}_{ij} U^* + \tilde{\pi}_{ij} L^* + \tilde{\pi}_{ij} U^*)}{|\tilde{A}^*|_w}, \quad (34)$$

**Step 4.** Rank all the alternatives $A_i$ $(i = 1, 2, \ldots, m)$ in accordance with the weighted projection $\Pr j_{\tilde{A}^*} A_i$. The larger the weighted projection $\Pr j_{\tilde{A}^*} A_i$, the closer the alternative
A_i is to the UIFIS $\hat{A}^*$, and the better the alternative, $A_i$. Therefore, all the alternatives can be ranked according to the values of the weighted projection so that the best alternative can be selected.

5. Illustrative Example

Now, we discuss a problem concerning with a manufacturing company, searching the best global supplier for one of its most critical parts used in assembling process. The attributes which are considered here in selection of five potential global suppliers, i.e., the set of alternatives is $A = \{A_1, A_2, A_3, A_4, A_5\}$, are: (1) $G_1$: overall cost of the product; (2) $G_2$: quality of the product; (3) $G_3$: service performance of supplier; (4) $G_4$: supplier’s profile; and (5) $G_5$: risk factor. An expert group is formed which consists of four experts from each strategic decision area. By statistical methods, the expert $e_k$ ($k = 1, 2, 3, 4$) evaluates the characteristics of the potential global supplier $A_i$ ($i = 1, 2, 3, 4, 5$) with respect to the attribute $G_j$ ($j = 1, 2, 3, 4, 5$) on the fuzzy concept “excellence”. Thus the four decision matrices $R^{(k)}$ ($k = 1, 2, 3, 4$) can be obtained and expressed in the Tables 1–4.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Expert 1 – intuitionistic fuzzy decision matrix $R^{(1)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.5, 0.4, 0.1)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.7, 0.3, 0)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.5, 0.4, 0.1)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.7, 0.2, 0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.4, 0.3, 0.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Expert 2 – intuitionistic fuzzy decision matrix $R^{(2)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.5, 0.5, 0)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.5, 0.3, 0.2)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.6, 0.2, 0.2)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.1, 0.8, 0.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Expert 3 – intuitionistic fuzzy decision matrix $R^{(3)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_1$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.5, 0.3, 0.2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.6, 0.3, 0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.7, 0.3, 0)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.7, 0.2, 0.1)</td>
</tr>
</tbody>
</table>
Considering that the attributes have two different types, we first transform the attribute values of cost type into the attribute values of benefit type by using (10), then $R^{(k)} = (r_{ij}^{(k)})_{m \times n}$ is transformed into $D^{(k)} = (d_{ij}^{(k)})_{m \times n}$, respectively, shown in the Tables 5–8.

Thus, we can utilize the proposed method to obtain the most desirable alternative(s). We first utilize (13) to determine the expert weights for each evaluation value and aggregate the different experts’ evaluations by (14). For example, the weights of the four experts for $A_1$ with respect to $G_1$ are $w_{i1}^{(1)} = 0.251, w_{i1}^{(2)} = 0.271, w_{i1}^{(3)} = 0.231, w_{i1}^{(4)} = 0.248,

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Expert 4 – intuitionistic fuzzy decision matrix $R^{(4)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>$G_2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.7, 0.2, 0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.4, 0.4, 0.2)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.4, 0.6, 0.0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Expert 1 – intuitionistic fuzzy decision matrix $D^{(1)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>$G_2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.3, 0.7, 0.0)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.2, 0.7, 0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.3, 0.4, 0.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Expert 2 – intuitionistic fuzzy decision matrix $D^{(2)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>$G_2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.5, 0.5, 0.0)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.5, 0.4, 0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.3, 0.5, 0.2)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.2, 0.6, 0.2)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.8, 0.1, 0.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Expert 3 – intuitionistic fuzzy decision matrix $D^{(3)}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>$G_2$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>(0.3, 0.5, 0.2)</td>
</tr>
<tr>
<td>$A_2$</td>
<td>(0.3, 0.6, 0.1)</td>
</tr>
<tr>
<td>$A_3$</td>
<td>(0.3, 0.7, 0.0)</td>
</tr>
<tr>
<td>$A_4$</td>
<td>(0.5, 0.4, 0.1)</td>
</tr>
<tr>
<td>$A_5$</td>
<td>(0.2, 0.7, 0.1)</td>
</tr>
</tbody>
</table>
respectively, and then we can obtain
\[
d_{11} = w_{11}^{(1)} d_{11}^{(1)} + w_{11}^{(2)} d_{11}^{(2)} + w_{11}^{(3)} d_{11}^{(3)} + w_{11}^{(4)} d_{11}^{(4)} = (0.43, 0.47, 0.1).
\]
In the same way, we can get all the other values to form the following collective decision matrix \( D = (d_{ij})_{5 \times 5} \) (see Table 9).

Assume the attribute weighting vector \( w = (0.16, 0.23, 0.18, 0.32, 0.12)^T \), then we employ (8) to calculate the weighted projection of the alternatives \( A_i \) on the IFIS \( A^* \):

\[
\Pr j_{A^*} A_1 = 0.301, \quad \Pr j_{A^*} A_2 = 0.300, \quad \Pr j_{A^*} A_3 = 0.285, \\
\Pr j_{A^*} A_4 = 0.320, \quad \Pr j_{A^*} A_1 = 0.307.
\]

Since
\[
\Pr j_{A^*} A_4 > \Pr j_{A^*} A_5 > \Pr j_{A^*} A_2 > \Pr j_{A^*} A_1 > \Pr j_{A^*} A_3.
\]

Then
\[
A_4 > A_5 > A_2 > A_1 > A_3,
\]
where “\( > \)” indicates the relation “superior to” or “preferred to”, hence, the most desirable alternative is \( A_4 \).

From the above example, we can see that, the weights of the experts can be deduced from the decision matrices and different attributes have different weights. This can avoid the affect of unfair evaluation values. For example, the weights to \( r_{11}^{(1)} = (0.4, 0.5, 0.1) \), \( r_{11}^{(2)} = (0.5, 0.5, 0.0) \), \( r_{11}^{(3)} = (0.3, 0.5, 0.2) \) and \( r_{11}^{(4)} = (0.5, 0.4, 0.1) \) are \( w_{11}^{(1)} = 0.251 \),

### Table 8

<table>
<thead>
<tr>
<th></th>
<th>( G_1 )</th>
<th>( G_2 )</th>
<th>( G_3 )</th>
<th>( G_4 )</th>
<th>( G_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.5, 0.4, 0.1)</td>
<td>(0.3, 0.6, 0.1)</td>
<td>(0.6, 0.2, 0.2)</td>
<td>(0.4, 0.5, 0.1)</td>
<td>(0.2, 0.7, 0.1)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.2, 0.7, 0.1)</td>
<td>(0.4, 0.6, 0)</td>
<td>(0.2, 0.7, 0.1)</td>
<td>(0.6, 0.2, 0.2)</td>
<td>(0.4, 0.5, 0.1)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(0.4, 0.4, 0.2)</td>
<td>(0.2, 0.8, 0)</td>
<td>(0.6, 0.2, 0.2)</td>
<td>(0.5, 0.4, 0.1)</td>
<td>(0.1, 0.9, 0)</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>(0.5, 0.4, 0.1)</td>
<td>(0.7, 0.2, 0.1)</td>
<td>(0.6, 0.3, 0.1)</td>
<td>(0.3, 0.5, 0.2)</td>
<td>(0.5, 0.4, 0.1)</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>(0.6, 0.4, 0)</td>
<td>(0.5, 0.4, 0.1)</td>
<td>(0.2, 0.8, 0)</td>
<td>(0.5, 0.4, 0.1)</td>
<td>(0.6, 0.3, 0.1)</td>
</tr>
</tbody>
</table>

### Table 9

<table>
<thead>
<tr>
<th></th>
<th>( G_1 )</th>
<th>( G_2 )</th>
<th>( G_3 )</th>
<th>( G_4 )</th>
<th>( G_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(0.43, 0.47, 0.1)</td>
<td>(0.65, 0.27, 0.08)</td>
<td>(0.51, 0.28, 0.21)</td>
<td>(0.53, 0.35, 0.12)</td>
<td>(0.30, 0.62, 0.08)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(0.33, 0.60, 0.07)</td>
<td>(0.50, 0.29, 0.21)</td>
<td>(0.61, 0.28, 0.11)</td>
<td>(0.62, 0.22, 0.16)</td>
<td>(0.28, 0.60, 0.12)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(0.35, 0.52, 0.13)</td>
<td>(0.37, 0.56, 0.07)</td>
<td>(0.63, 0.23, 0.14)</td>
<td>(0.52, 0.29, 0.19)</td>
<td>(0.45, 0.51, 0.04)</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>(0.36, 0.51, 0.13)</td>
<td>(0.52, 0.32, 0.16)</td>
<td>(0.48, 0.44, 0.08)</td>
<td>(0.72, 0.23, 0.05)</td>
<td>(0.41, 0.44, 0.15)</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>(0.58, 0.30, 0.13)</td>
<td>(0.57, 0.37, 0.06)</td>
<td>(0.68, 0.22, 0.1)</td>
<td>(0.51, 0.40, 0.09)</td>
<td>(0.46, 0.42, 0.12)</td>
</tr>
</tbody>
</table>
of could rationally demonstrate the quality of an alternative; in other words, the lower values where familiar with the attribute w
r
nient to use in practical applications.

If we utilize the IFWA (Xu, 2007a) operator
d_i = IFWA(d_{i1}, d_{i2}, \ldots, d_{in}) = w_1d_{i1} + w_2d_{i2} + \cdots + w_n d_{in}, \quad i = 1, 2, \ldots, 5
(35)
to derive the collective overall preference value d_i (i = 1, 2, \ldots, 5) of the alternative A_i in the collective intuitionistic fuzzy decision matrix D = (d_{ij})_{5 \times 5}, where w = (0.16, 0.23, 0.18, 0.32, 0.12)^T is the weighting vector of the attribute, then we have
d_1 = (0.53, 0.35, 0.12), \quad d_2 = (0.52, 0.32, 0.16), \quad d_3 = (0.48, 0.38, 0.14),
d_4 = (0.56, 0.34, 0.10), \quad d_5 = (0.57, 0.33, 0.10).

To rank the IFN, several relevant approaches have been presented, see, for instance, Chen and Tan (1994), Hong and Choi (2000), Liu and Wang (2007), Xu (2007a), and Szmidt and Kacprzyk (2008, 2009). Among the existing ranking techniques, Szmidt and Kacprzyk’s (2008, 2009) new ranking approach for IFNs takes into account not only the amount of information (both positive and negative) associated with an alternative but also the reliability of information represented by an alternative indicating how sure the information is, we may use it here to evaluate the IFN d_i (i = 1, 2, \ldots, 5), i.e.,

S_{IFN}(d_i) = 0.5(1 + \pi_i)d_{IFN}(d_i, \alpha_i^*)
(36)

where \pi_i is the intuitionistic fuzzy index (or hesitation margin) of d_i, \alpha_i^* = (1, 0, 0) is the ideal positive intuitionistic fuzzy alternative, and d_{IFN}(\cdot, \cdot) denotes the normalized Hamming distance between IFNs, i.e.,
d_{IFN}(\alpha_1, \alpha_2) = \frac{1}{2}(|\mu_{\alpha_1} - \mu_{\alpha_2}| + |v_{\alpha_1} - v_{\alpha_2}| + |\pi_{\alpha_1} - \pi_{\alpha_2}|)
(37)

where \alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1}, \pi_{\alpha_1}) and \alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2}, \pi_{\alpha_2}) are two IFNs.

Szmidt and Kacprzyk (2008, 2009) pointed out that the measure S_{IFN}(d_i) from (36) could rationally demonstrate the quality of an alternative; in other words, the lower values of S_{IFN}(d_i), the better the corresponding alternatives A_i.
Finally, use (36) to calculate the measure $S_{IFN}$ of $d_i$ ($i = 1, 2, \ldots, 5$) as follows:

- $S_{IFN}(d_1) = 0.267$
- $S_{IFN}(d_2) = 0.275$
- $S_{IFN}(d_3) = 0.295$
- $S_{IFN}(d_4) = 0.240$
- $S_{IFN}(d_5) = 0.236$

According to $S_{IFN}(d_i)$ ($i = 1, 2, \ldots, 5$), we can easily rank all of the alternatives: $A_5 \succ A_4 \succ A_1 \succ A_2 \succ A_3$, and hence, the best choice is $A_5$. As we can see, depending on different methods used, the results may be different.

Similarly, if the preferences given by the experts are expressed in interval intuitionistic fuzzy decision matrices, then we can utilize the Algorithm 2 to derive the weights of the experts.

**6. Conclusions**

The existing MAGDM approaches based on IFS or IVIFS can only cope with the situation that the weights of the experts are determined beforehand and the weights of the decision makers are the same for all the attributes. In this paper, we have developed a new algorithms in which the weights of the experts are derived from the decision matrices and decision makers have different weights for different attributes. The closer an evaluation value is to the ideal decision of the evaluation values, the larger the weight is, and the farther an evaluation value is from the ideal decision of the evaluation values, the smaller the weight is. These can effectively avoid the unreasonable evaluation values due to the lack of knowledge or limited experience of experts. Thus, the results are more reasonable.

Then, we develop a straightforward and practical algorithm to rank alternatives based on the weighted projection of the alternative on the intuitionistic fuzzy ideal solution (IFIS). Furthermore, we have extended the developed models and procedures to deal with the MAGDM with interval-valued intuitionistic fuzzy information. Finally, numerical examples are given to further illustrate the practicality and efficiency of the new algorithms. Further studies could be aimed at developing hybrid decision making methods based on different linguistic representation methods (e.g. 2-dimensional uncertain linguistic information (Liu, 2012)) and fuzzy multi-criteria decision making methods (Zavadskas et al., 2012).

**Acknowledgements.** This paper is supported by the National Funds of Social Science of China (Nos. 12ATJ001, 12&ZD211), Science Foundation of Ministry of Education of China (No. 13YJA910001), the Key Research Center of Philosophy and Social Science of Zhejiang Province – Modern Port Service Industry and Creative Culture Research Center, the MOE Project of Key Research Institute of Humanities and Social Sciences in Universities (No. 13JD910002), Zhejiang Provincial Key Research Base for Humanities and Social Science Research (Statistics), Projects in Science and Technique of Ningbo Municipal (No. 2012B82003), Zhejiang Province Natural Science Foundation (No. Q12G030068), Ningbo Natural Science Foundation (No. 2013A610286) and Projects of Federation of Social Sciences Research in Zhejiang Province (No. 2013B069).
References


S.Z. Zeng graduated from the Tianjin University and obtained the master degree in applied mathematics in 2007. At present, he is studying his in-service doctor of applied statistics at Zhejiang Gongshang University. He has published more than 40 papers in journals, books and conference proceedings including journals such as Statistics Research, Economic Research, Knowledge-Based Systems and Group Decision and Negotiation. His main research fields are aggregation operators, decision making, comprehensive evaluation, and uncertainty. Now he is a full-time lecturer at Zhejiang Wanli University.

T. Baležentis is a Specialist at the Lithuanian Institute of Agrarian Economics. He received Student scientific paper award (2011) from the Lithuanian Academy of Sciences and President of the Republic of Lithuania A. Brazauskas scholarship for 2012–2013. T. Baležentis has published over 40 peer-reviewed papers on multi-criteria decision making, benchmarking, and agricultural economics.

J. Chen graduated from the Zhejiang Gongshang University and obtained the doctor degree in applied economics in 2010. He has published more than 30 papers in journals, books and conference proceedings including such journals as Statistics Research, Economic Research and Decision Making. His main research fields are group decision making and multi-attribute decision making. Now he is an associate professor of applied statistics at Zhejiang Gongshang University.

G.F. Luo graduated from the Zhejiang Gongshang University and obtained a bachelor degree in applied economics in 2011. At present, she is studying her doctor degree in statistics at Zhejiang Gongshang University. Her main research fields are decision making, comprehensive evaluation, and income distribution.

Projekcijos metodas grupiniam daugiakriteriniam sprendimų priėmimui su intuityvia neraiškiąja informacija

Shouzhen ZENG, Tomas BALEŽENTIS, Ji CHEN, Gangfei LUO

Straipsnyje nagrinėjamos intuityviaus neraiškiojo sprendimų priėmimo problemos, kuomet informacija apie nagrinėjamas alternatyvas yra išreiškiama intuitivyiais neraiškiaisis skaičiais, o ekspertams nustatomi svoriai iš anksto nėra žinomi. Pasiūlyti ekspertams nustatyti svorius atitinkamų įverčių įvertinimų projektacijos į idealų įvertinimų įverčį. Taigi, ekspertams suteikiama didesnė svarbos, jei jų pateikta įvertinimų įverčiai labiau sutampa su idealizuoto įverčiu, ir atvirkščiai. Tuomet nagrinėjamos alternatyvos ranguojamos pagal jų projektacijos į idealų įverčių įvertinimą. Minėtas algoritmas pritaikytas ir intervaliniams intuitivyiaus neraiškiojų duomenims. Pateiktas praktinis sprendimų priėmimo pavyzdys atskleidžia metodą praktiškąją ir veiksmingumą.