Visualization of Zetabulbs: Mandelbulbs associated with the Riemann zeta function

Lukas Kuzma, Martynas Sabaliauskas, Igoris Belovas

E-mails: igoris.belovas@mif.vu.lt, martynas.sabaliauskas@mif.vu.lt, lukas.kuzma@mif.vu.lt

Vilnius University, Institute of Data Science and Digital Technologies

Abstract

This research delves into Mandelbulbs associated with the Riemann zeta function, merging informatics, mathematics, and informatics engineering. It pioneers the systematic exploration of this intricate connection, addressing scientific problems with efficient algorithms for zeta-function calculations and parallel algorithms for Zetabulb visualizations.

Introduction

The aim of this work is to generate and explore Mandelbulbs associated with the Riemann zeta function (Zetabulbs). The main tasks are:

- Implementation of GPU-based version of the modified Borwein algorithm for the computation of the Riemann zeta function.
- Construction and realization of efficient algorithms for the real-time generation of the Mandelbulb images.
- Association of the Mandelbulbs to the Riemann zeta function.
- Visualization of the Zetabulbs.
- Exploration of newly received fractal structures.

Let $s \in \mathbb{C}$. The Riemann zeta function is defined for $\sigma > 1$ by the Dirichlet series or the Euler product,

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right)^{-1},$$

and by analytic continuation elsewhere, except for the pole at $s = 1$.

Three dimensional Mandelbrot set

In 1997 Jules Ruis discovered a method (which was later improved by Paul Nylander and Daniel White), to visualize an interpretation of the three-dimensional Mandelbrot set using spherical coordinates.

$$v^n = r^2 \langle \sin(\theta n)\cos(\phi n), \sin(\theta n)\sin(\phi n), \cos(\theta n)\rangle,$$

where

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \arctan(\sqrt{x^2 + y^2}/z), \quad \phi = \arctan(y/x).$$

Let **SDFMandelbulb**(s, p, k) be a signed distance function of a Mandelbulb. Here s - the three-dimensional coordinates of the origin point, p - the power of the Mandelbulb, k - the maximum number of iterations.

Definition of the Zetabulbs

The association of the complex plane and the Cartesian coordinate system can be accomplished in two steps. First, we compute the Riemann zeta function value z for an argument somehow connected to the point c. Next, we construct a new point s and compute the signed distance function of a Mandelbulb for this point. An example of this association is presented in following algorithm. Let s - the three-dimensional coordinates of the origin point, p - the power of the Mandelbulb, k - the maximum number of iterations, c - complex number used to associate the Mandelbulb with the real and imaginary surfaces of the Riemann zeta function. The signed distance function can be expressed as follows:

def **SDFZetabulb**(s, p, k, c):
  $z \leftarrow \zeta(\Re c + s_x + i(\Im c + s_y))$,
  $s \leftarrow (\Re z, \Im z, s_z)$,
  return SDFMandelbulb(s, p, k).

Conclusion and discussion

Figures 1 and 2 has been generated using our implementation.

- Can you find similarities and differences between them?
- Does the figure 1 resemble any Hollywood movie?

Visualizations of the Mandelbulbs and Zetabulbs

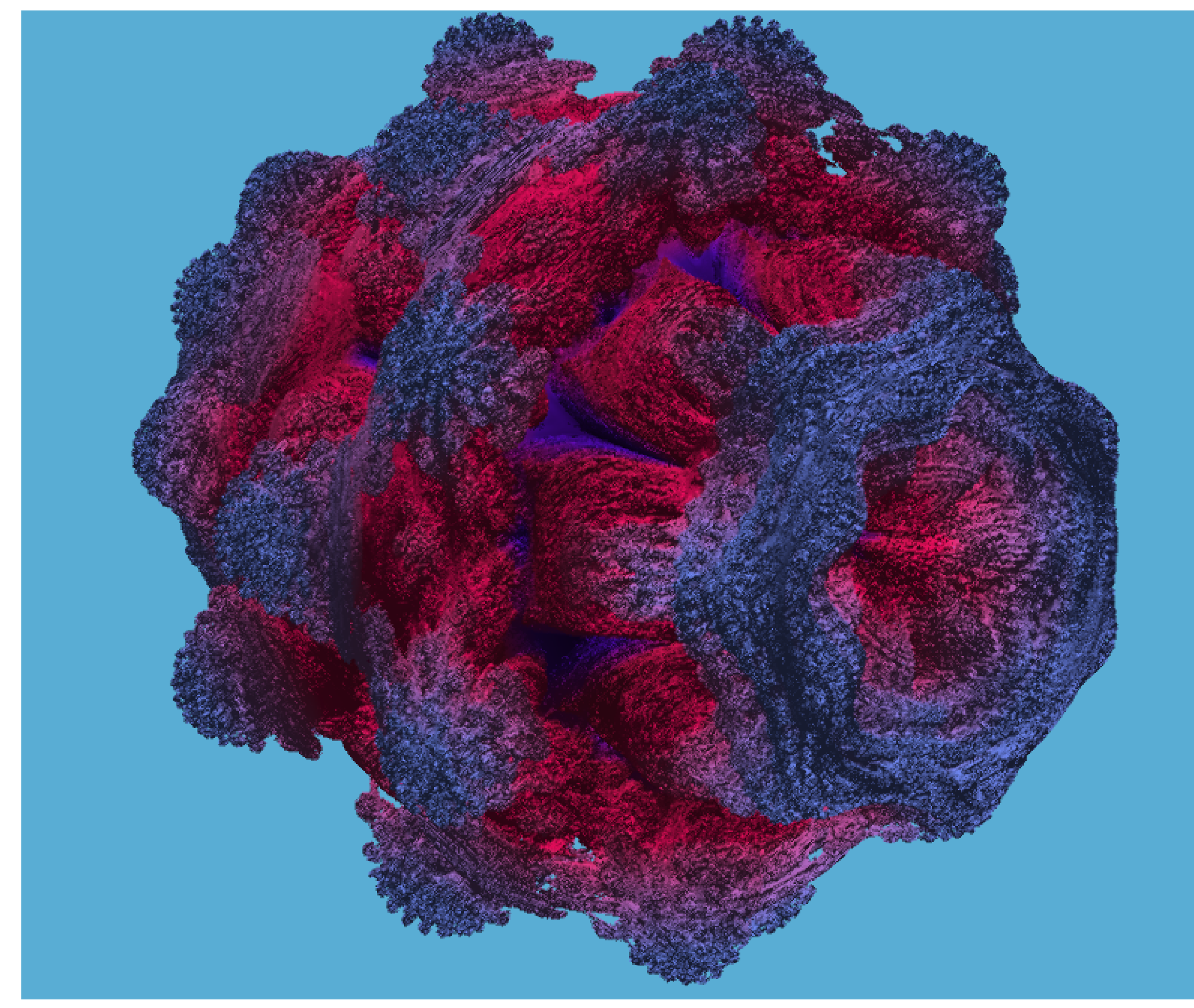

Figure 1: 8^{th} degree Mandelbulb

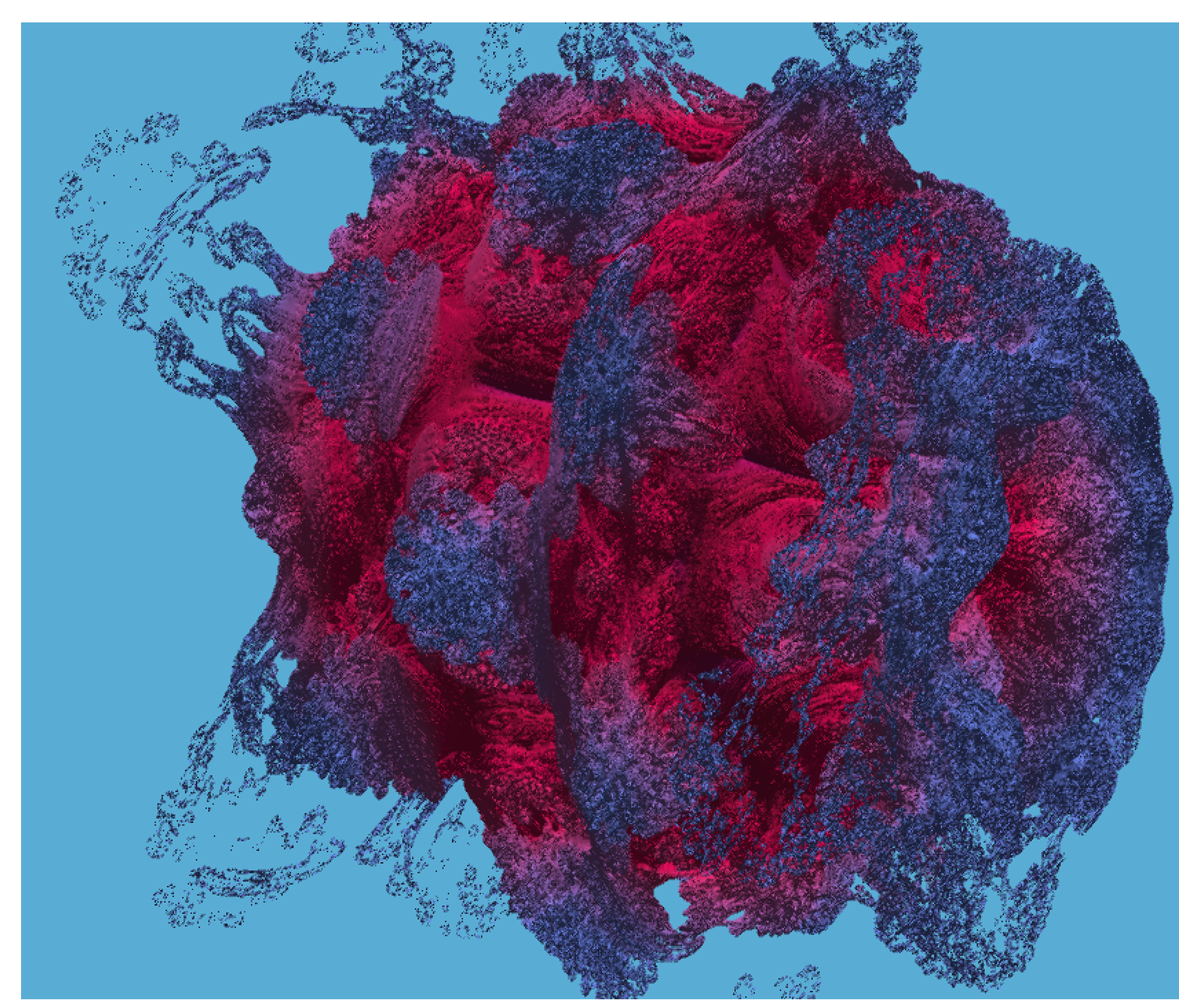

Figure 2: Zetabulb centered on the first non-trivial zero

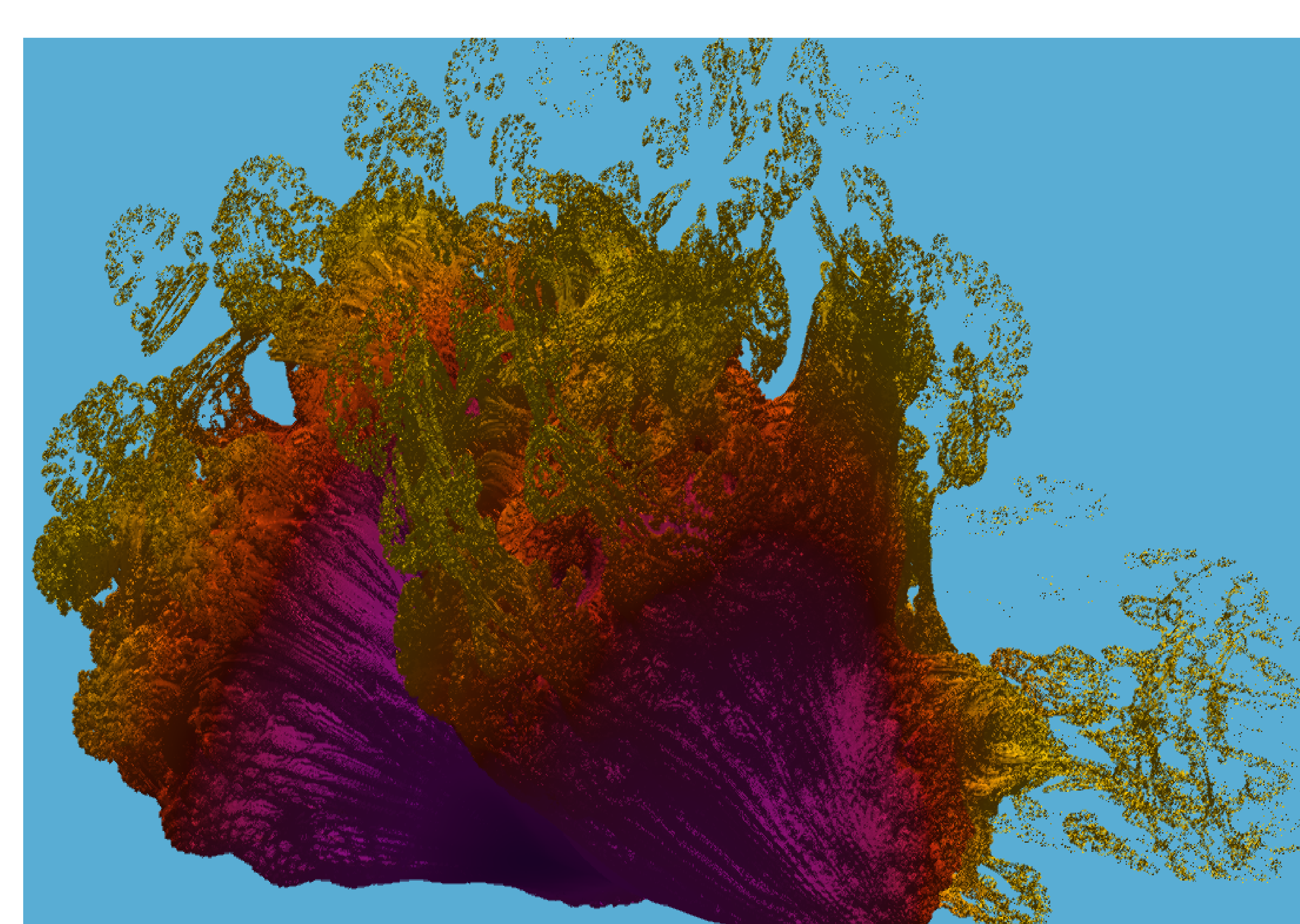

Figure 3: Closeup of a Zetabulb with $c = (1.5 + 75.65i)$