Constructing Production Planning Tasks: A "Teltonika" Case Study

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Introduction

Production planning is a pivotal domain that determines the efficiency and competitiveness of modern enterprises in today's market. The aim of this presentation is to discuss how mathematical modeling can be applied in the construction of production planning tasks, using the "Teltonika" case as an example.
In the main formulation of the production planning task, critical parameters such as the number of orders, quantity of products, number of workers in each stage, and the number of stages were identified. These data provide a profound insight into the timelines of the production process, product delivery deadlines, and distribution of workers.
The detailed formulation of the task incorporated mathematical nuances, like binary variables and time parameters. Mathematical modeling techniques were employed to describe the production process in the most accurate manner, facilitating a better understanding and analysis of production workflows.
The essence of the presentation is to highlight how a detailed task formulation is vitally important for the proper understanding and analysis of production processes. Emphasis is placed on how mathematical structures and methods enable a precise and comprehensive description of the production process.

Formalization of the Manufacturing Process

Each order u is defined by product code c_u and amount of units Q_{u,c_u}. Each code goes through certain stages of the production process, with certain work being carried out at each stage until the product is manufactured. Therefore, the manufacturing process can be described by stages $\mathrm{k} = 1, \dots, 7$: labeling, SMT, soldering, depaneling, testing, assembly, and packaging. Each stage consists of workers $j = 1, \dots, J_k$, who can perform the tasks of this stage. Each job is defined by the code i and the worker $j < J_k$, performing this job, i.e., in reality, different workers can do the same job, but the performance speed may vary, so during modeling, we will consider that these jobs are different.
The manufacturing process of each product could be simplified and represented by a graph (see Figure 1), where the vertices are the workers, and each connection between the stages is expressed by edges $(j, \tilde{j})$, $j < J_k, \tilde{j} < J_{k+1}$. Each code i has a unique set of edges, i.e., in general, the graphs for different codes are different.
Creating a production plan, consisting of orders $u < U$, which are made up of different codes k_i, we have to consider these requirements:

- a product, i.e., code i, visits only one worker at each stage;
- each worker $j < J_k$ can perform only a certain set of jobs, which depends on the code.

In the modeling of production processes, it's essential to represent the potential interactions among various products that are manufactured concurrently. This means that while the vertices of the graph are shared, the graphs for different products differ due to distinct edges. The existence of these edges is contingent upon worker constraints. Given that this is a directed acyclic graph (DAG) and at each stage, a specific product is assigned to only one worker, it isn't practical to model the entire graph. Therefore, it is more efficient to organize vertices by stages, thereby directly applying the concept of stages from the actual production process.

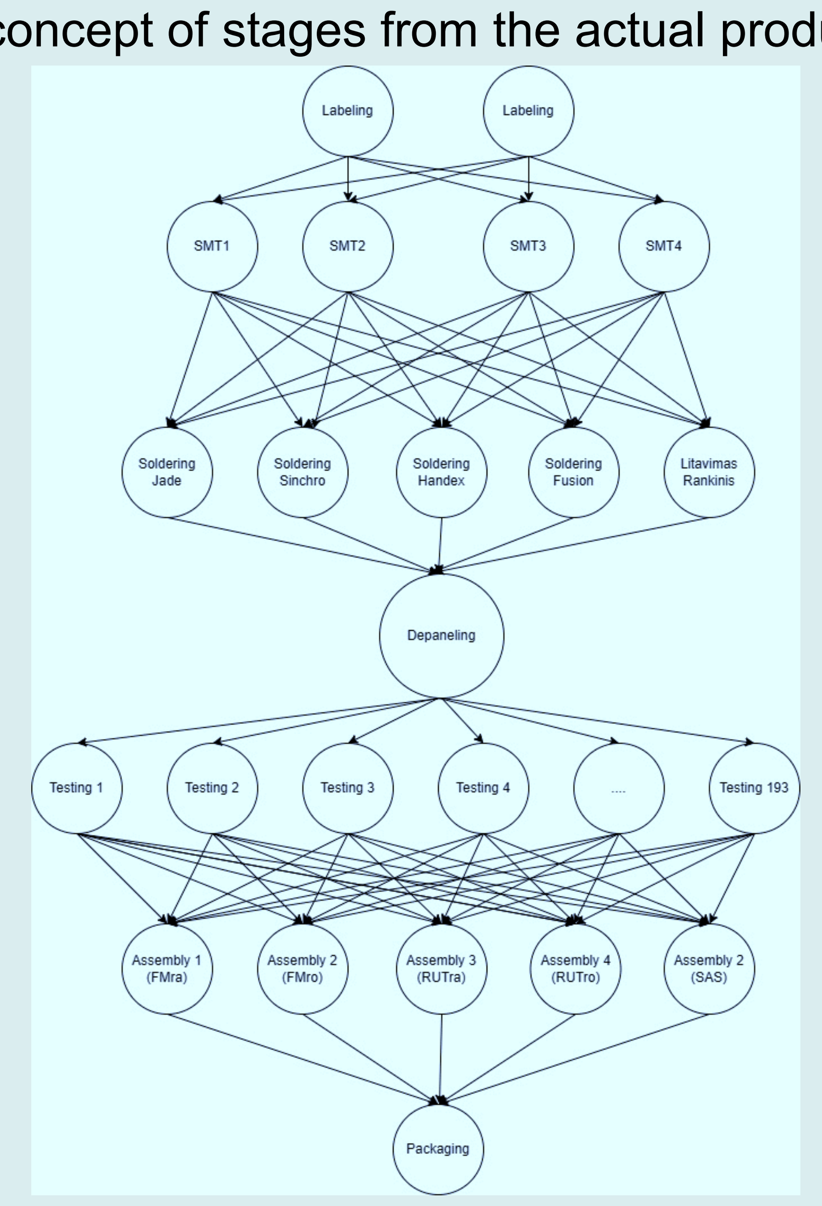

Figure 1. The graph of manufacturing process in the case when the code has all possible edges

Problem Formulation

Data:

U : number of orders; I: number of products; I_u: set of products included in the u-th order; J_k: number of workers at stage k; K: number of stages (in this case 7); T_u: deadline for order u; $\mathrm{Q}_{u,i}$: quantity of product i in order u; $\mathrm{t}_{i,j,k}$: processing time of product i by worker j at stage k.

Variables:

$\mathrm{F}_{u,i,j,k,n}$: binary variable indicating whether worker j a starts processing product i, which is the $n-th$ order position in order u, at stage k. Values 1 and 0 mean ”yes” and ”no”, respectively.
$\mathrm{S}_{u,i,j,k,n}$: time when worker j starts processing product i, which is the $n-th$ order position in order u, at stage k. S is non-negative only where $\mathrm{F}_{u,i,j,k,n} = 1$, elsewhere it equals -1.
$L_{u,i,j,k,n}$: time when worker j finishes processing the $n-th$ product i at stage k according to order u.

Objective Function:

$$\min_{FS} \sum_{i=0}^{n} P\left(\max_{\substack{i=1,..,I_u\\ n=1,\dots,Q_{u,i}}} L_{u,i,K,n}, T_u\right), \tag{1}$$

$$FS = (\{F_{u,i,j,k,n}, \forall u < U, \forall i \in I_u, \forall n \leq Q_{u,i}, \forall k < K, \forall j < J_k\},$$
$$\{\mathrm{S}_{u,i,j,k,n},\ \forall u < U, \forall i \in I_u, \forall n \leq Q_{u,i}, \forall k < K, \forall j < J_k\}).$$

P – is a functional that allows evaluating the correspondences of the first and second arguments, its simplest version would be the modulus of the difference.

Constraints:

Each product must be processed at each stage:

$$\sum_{j=1}^{J_k} F_{u,i,j,k,n} = 1 \qquad \forall u < U, \forall i \in I_u, \forall n \leq Q_{u,i},\ \forall k < K. \tag{2}$$

Product processing start at stage k cannot begin earlier than it ends at stage $k\ -1$:

$$S_{u,i,j,k,n} \geq L_{u,i,j_{k-1},k-1,n}\ \forall u < U, \forall i \in I_u, \forall n \leq Q_{u,i},\ \forall k < K. \tag{3}$$

Two products cannot be processed at the same time by the same worker:

$$S_{u,i,j,k,n} \geq L_{u',i',j,k,n'} \text{ or } S_{u',i',j,k,n'} \geq L_{u,i,j,k,n},\ \forall u, u', i, i', j, k, n, n'. \tag{4}$$

A case study to start the analysis might look like this

$$T(u) = \min_{FS} \max_{\substack{1\leq i\leq I_u\\ 1\leq n\leq Q_{u,i}}} L\big(j(i,n,k)\big), \qquad u = 1, \dots, U.$$

Solving this problem would give lower bounds for the more general problem.

Conclusions

Mathematical modeling applied to Teltonika's production planning reveals the efficacy of quantitative methods in refining manufacturing processes. Key variables including order sequencing, product progression, and labor distribution are central to this model, providing a robust framework for effective timeline management and workforce optimization.
The study's key insights include:

1. **Process Efficiency**: Through binary variables and time parameters, the model offers an intricate portrayal of production, aiding bottleneck identification and workflow optimization, thus bolstering efficiency.
2. **System Adaptability**: The model's capacity to adapt to variations in orders, products, and labor highlights its flexibility, crucial for responding to market and operational shifts.
3. **Forecasting and Risk Management**: By quantifying production aspects, the model enables predictive planning and risk management, essential for continuous production and deadline adherence.
4. **Labor and Stage Management**: Focusing on labor distribution and processing times ensures balanced workload distribution, optimizing performance and ensuring well-timed production stages.