

VILNIUS UNIVERSITY

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**INTERACTIVE VISUALIZATION MODEL FOR THE CONSTRUCTIONIST  
TEACHING AND LEARNING OF GEOMETRY**

Summary of Doctoral Dissertation  
Physical Sciences, Informatics (09 P)

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VILNIAUS UNIVERSITETAS

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**INTERAKTYVAUS VIZUALIZAVIMO MODELIS GEOMETRIJOS  
KONSTRUKCIONISTINIAM MOKYMUUI IR MOKYMUISI**

**Daktaro disertacijos santrauka**

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## 1. General characteristics of the dissertation

When computers appeared in schools some domains of informatics and education became very close [Heb86]. Sometimes they intertwined so strongly that it was difficult to say where one ended and another began. The topic of the paper falls in the intersection of informatics and mathematics education research fields. If we aim to develop any education tool we need to analyse the area of practical usage of this tool. Therefore, the aspects of psychology, pedagogy, and mathematics education have to be taken into account. The core research problem as well as novelty lies within informatics theory domain (Fig. 1).

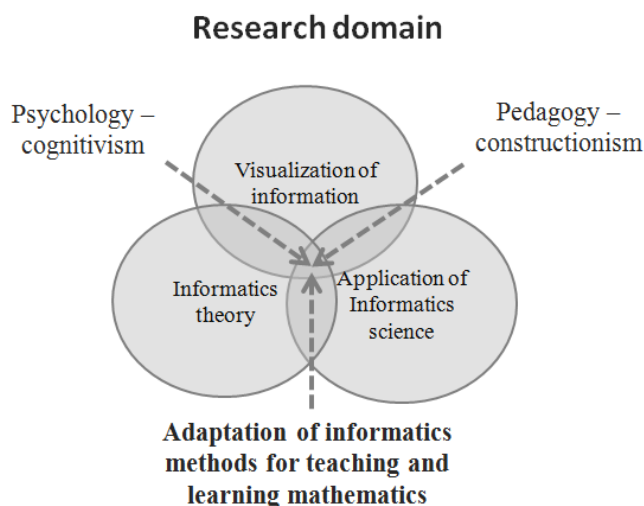


Figure 1. The problem domain

### 1.1. Scope and relevance

It is said that a picture conveys more than a thousand words. We can express the intricate mathematical proof or other things of value in a geometric drawing [Cas00]. Since the Babylonian times visualization helped mathematical phenomena to survive and spread. Modern dynamic graphics, computer programs and other educational technology influenced visualization and teaching of geometry [Arc03] [Boz05]. Teaching and learning approaches of mathematics, as well as geometry are changing [HHK08]. The information technologies became an important integral part of any subject didactics. Didactic triangle suggested by H. Steinbring [Ste05] was extended into the didactic tetrahedron, where one more element was added. The triangle became a tetrahedron of four elements: a teacher, a student, mathematics, and information technology. Each face of didactic tetrahedron showed the relationships between the teacher, the student's mathematical knowledge and technology [OMH10].

Education of mathematics has been increasingly focused on the interactive learning methods. The scientific literature shows that interactive teaching of mathematics goes parallel to dynamic sketches, and there are more than 40 tools of dynamic geometry all over the world [AEB10] [LAS08] [JQ07]. They all have the same goal – to teach geometry using the dynamic sketches while helping the students gain an insight into geometry (and sometimes algebra) concepts and proofs. Those researchers who can be considered constructionists see an important opportunity in these programs t. e. geometry is studied by constructing drawings. Educational and psychological research shows that

learning by constructing dynamic sketches helps to understand geometry deeper, the understanding of geometric concepts becomes more meaningful, geometric thinking and geometric imagination are developed [Pat08].

Teachers of mathematics usually tend to believe and emphasise the benefits of interactive (dynamic) geometry for learning. The teachers, however, still rarely use these technologies. The commonplace reason is the lack of technical skills this preventing effective creation of new samples.

A long-term observation inspired to develop an approach of making the studies of mathematics easier both for the students and teachers. Therefore, the aim of the research is to construct a model of interactive visualization for secondary school geometry. The model should help construct interactive objects, using dynamic geometry system, and direct teachers and students to master their skills in a dynamic geometry.

Dynamic geometry can be presented as a specific programming application. Most of teachers of mathematics have difficulty when creating interactive microworlds using dynamic geometry systems. An informatics problem can be recognised as projection of a geometric topic into the interactive microworld by using dynamic geometry. The operations and objects of dynamic geometry can be specified by abstract data types (ADT). ADTs are defined by using heterogeneous algebra. New ADTs are applied in creating the microworlds.

## **1.2. The research object**

In this dissertation, the object of the research is the adaptation of informatics methods to interactive visualization

## **1.3. Objective and tasks**

The objective of the research is to develop an interactive visualization model based on dynamic geometry for constructionist teaching and learning of geometry.

In order to achieve the objective, the following tasks have to be solved:

1. To explore informatics methods for improving teaching and learning of geometry by using constructionist approach.
2. To create a model of interactive visualization.
3. To define interactive visualization criteria.
4. To adapt the abstract data type theory for modelling scenarios of interactive visualization.
5. To implement the model and perform its validation

## **1.4. Methods of research**

In the dissertation, various research methods, theories and formalisms were used, such as the analysis of scientific literature, use of constructive methods to develop the model, implementation of abstract data type theory of formalization of different scenarios, homogeneous and heterogeneous algebra theories for specifications of ADTs, informal pedagogical methods and pedagogical theories (mainly constructionism), a quasi-experiment and statistical analysis of its results. Descriptive statistics and t-test statistical analysis for two independent samples have been used to analyse the data of the

experiment.

### **1.5. Statements presented for defence**

1. The abstract data type theory can be applied to create scenarios of interactive microworlds for teaching and learning geometry.
2. The developed interactive visualization model can help to create a digital tool for teaching and learning geometry and other topics of mathematics.
3. The developed interactive visualization model for teaching and learning geometry can be implemented in various dynamic geometry systems.
4. Dynamic geometry system is an effective tool for teaching and learning geometry by using constructionist methods.

### **1.6. Scientific novelty**

The scientific novelty of this research can be seen in several areas:

1. *In Informatics*: the abstract data type theory is adapted for the formalization and realization of the interactive visualization model. The ADT syntax and semantics is formalized. The ADT model, based on the technological possibilities of dynamic geometry systems is developed.
2. *In informatics engineering*: the interactive visualization model for teaching and learning geometry is developed. A specified criteria model for creating digital a geometry learning tool is composed.
3. *In pedagogy*: The *Baytak* model of the constructionist teaching and learning of mathematics by using dynamic geometry system is adapted and extended.

### **1.7. Practical relevance**

The practical relevance can be considered in several aspects:

1. The formalization of ADT syntax and semantics of the technological possibilities of dynamic geometry system could be used to automate creation of dynamic sketches in the future works.
2. The extended *Baytak* constructionist teaching and learning model provides teachers with an effective use of dynamic geometry system by employing constructionist methods in their lessons.
3. The interactive visualization quality criteria model helps to evaluate or qualitatively visualize topics of mathematics.
4. The model of interactive geometry visualization is applied to visualize 9<sup>th</sup> and 10<sup>th</sup> grade topics of mathematics within the extent of plane geometry, functions, equations and systems of equations.

### **1.8. Approbation of the research results**

The main results of the dissertation are represented in ten scientific publications – seven in scientific periodical, three in international conference proceedings.

## **1.9. The structure and scope of the dissertation**

The dissertation consists of an introduction, four main chapters and the conclusion. A list of author publications, a list of references and seven appendixes are given additionally. The total volume of the dissertation consists of 140 pages, including 35 figures, 10 tables and 134 references.

## **2. Methods of informatics applied for the constructionist teaching and learning of geometry**

### **2.1. The concept of constructionist teaching and learning**

Education is affected by various psychological theories. For many years it has been based on the ideas of behaviourism and focuses on the accumulation of knowledge, teaching lectures, and reflection. At the same time, other psychological trends have been developed, i.e. a cognitive theory which has changed the approach to teaching. Teaching ideas based on the collection, processing, development are directed to thinking. Basically the directions of both the teacher's and student's roles are different – in the first one the teacher is a trainer and information provider, whereas the student is the receiver of information. In the second one, however, the teacher is an advisor while the student is the information gatherer, handler and developer [Hub00].

Therefore, the present-day teachers of mathematics use quite a comfortable method which does not require a lot of preparation for teaching, i. e. lecture. Otherwise, the teacher seeks to integrate cognitive ideas and methods, based on looking at the student as a thinking person, i. e. through discussions, problem solving, and collaboration. Thus, both behavioural and cognitive methods of education are adequate when they match teaching and learning goals.

With the increase in information contents, the man cannot but is forced to remember a superfluity of information. Thus, Jean Piaget's psychology-based training – constructivism – is used the main principle of which is experience-based knowledge creation, when the learner is actively involved in teaching and learning [Pia54]. Shifting from teaching to learning shows that the whole educational process turns its focus to learning and new tools that encourage everybody to learn successfully and be motivated. The teacher, in essence, becomes a counsellor who helps to strengthen the links between different areas of education.

Seymour Papert [Pap93] examined the topic in more detail and wrote that it was important how children learn in a particular context using objects created on their own and by others. They focused on the role of ICT in human learning. The new learning theory – constructionism – thus emerged where the main point is to learn different methods and ways of purposeful information while selecting and absorbing from the abundance of knowledge, and using it effectively to create new knowledge.

A. Baytak argues that the constructionist learning consists of two steps: the internal step when learning is an active process and students construct their knowledge from their experiences, and the external one which is based on the research and it suggests that students learn best by making artefacts that can be shared with others [Bay11]. The internal step derives from constructivism, the external step relates to constructionism.



## 2.2. Baytak model of constructionist learning

A. Baytak presents the model of constructionist learning, where he declares four steps: planning, designing, testing, and sharing [Bay11]. His model is developed for learning by game design. Internal and external learning are distinguished. He attributes the internal learning to constructivism and the external learning to constructionism ( Fig. 2).

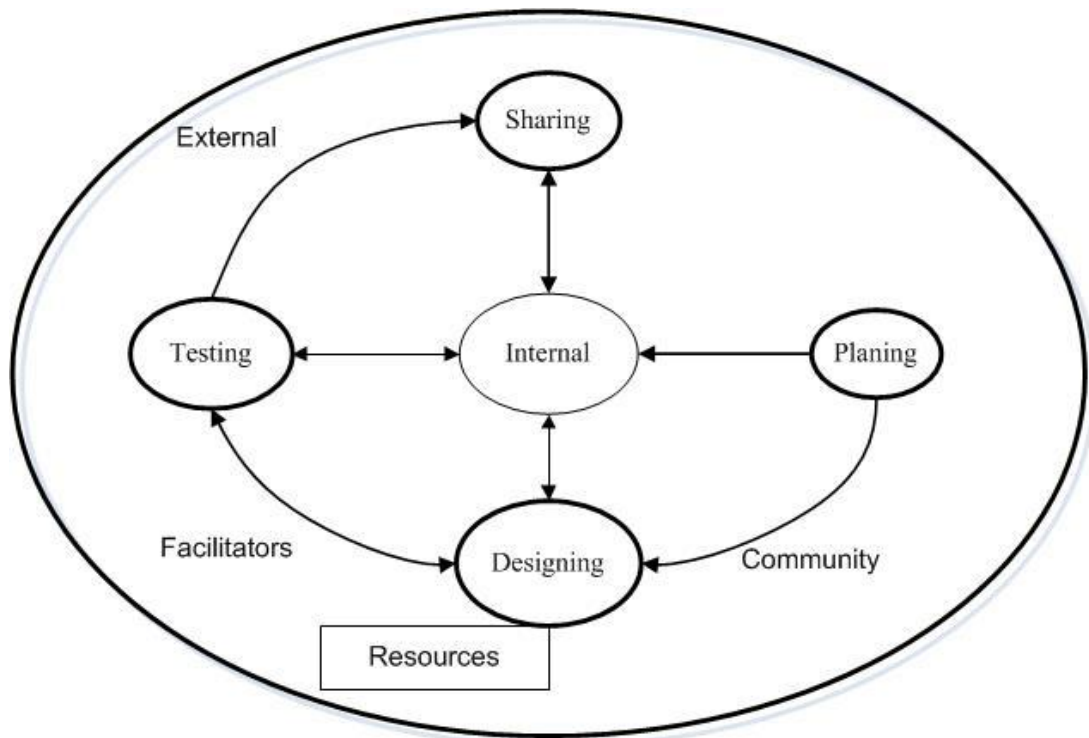


Figure 2. Baytak model for constructionist learning

## 2.3. Information technologies in mathematics teaching and learning

The existing literature on students' conceptions of mathematics indicates that students of all grades have difficulties with understanding geometric sketches. To improve mathematical skills of all students in a classroom is one of the most difficult problems for teachers of mathematics. Information technology is one of the modern tools which can help both teachers and students in the learning process.

The new psychological tendency and development of information technologies have affected the development of new technological tools for using active teaching and learning methods. The first ideas of constructionism based on information technology in education have been promoted by S. Papert mentioned above. He is the creator of the education technology, based on the constructionist idea, i. e. learning by doing. Most of the follow-up educational software takes root in S. Papert ideas and methodologies. A dynamic geometry is one of such tools for both teachers and students. A dynamic geometry is aimed to improve students' geometric skills and to make their knowledge deeper and more significant.

## 2.4. Interactive geometry visualization in dynamic geometry systems

There are many definitions of dynamic geometry in scientific literature [Jac04] [GJ96] [Oli00] [FAS10]. A review of those different definitions of dynamic geometry

will author present the view of dynamic geometry. Dynamic geometry is a technological tool developed to construct directly related geometric objects of a mathematical phenomenon which can be transformed and explored using a variety of technological and computerized management tools while, at the same time, maintaining the relationships between the designed objects.

The fundamental idea of dynamic geometry lies in the theory of constructionism – students learn by their own experience and share with others. Dynamic geometry is constructed for teaching and learning geometry (and partially algebra) in such a way that it helps to use various methods of teaching and learning, makes the teaching process more attractive, and learns geometry deeper. Dynamic geometry is designed so that the student is actively involved in the design and study.

Studies about the use of information technologies (IT) in education show that the teachers rarely employ dynamic geometry in class. Some studies show that the one of the aspects of this phenomenon is a teacher's view of the educational process [SK11].

The low digital competency of the teachers of mathematics has great impact on the usage of dynamic geometry for teaching mathematics [SK11]. Such ideas can be found in a few scientific sources [Jac93] [HHL09].

Another important concept in this research is 'visualization' (spatial thinking, visual image). A few studies offer different interpretations of the concept [War12] [ZC91] [Pre89] [Bis88] [Boz05]. The visualization is based on making visible invisible. For example, a concept, axiom or theorem can be visualized in mathematics because their expressions in words can be to more difficult to understand. The concept of visualization in scientific literature can be defined as a verbal noun which implies process and action or as a noun which implies vision or entity [Bis88].

The concept of interactive (dynamic) geometry visualization describes an action when the opportunity to move geometric figures exist usually on computers, but it could be done in mind too with trained imagination [Boz05].

The object of this research is close to W. Zimmermann and S. Cunningham's definition – interactive geometry visualization is the usage and creation of geometric concepts, axioms, theorems and problems in dynamic geometry system [ZC91].

## **2.5. Abstract data type**

The theory of ADT is used to reduce and specify software by defining abstract data values. These values are generated by functions and constructs which are defined by the first line direct equations [JO97]. Three requirements for ADT are presented by [LG86]:

- All functions of values of ADT have to be defined in the description of data type.
- The user of ADT may not know how ADT values are stored in the memory of a computer.
- The user of ADT can only use functions of ADT with ADT values but cannot operate with the reflections of those values in the memory of a computer.

A new ADT has to be described by a formal definition and implementation [Gut87]. ADT can be described by informal and formal methods. In order to avoid ambiguity, only formal methods are used to describe ADT. The syntax specification defines the names, domains, and ranges of the operation type. The semantic specification contains a set of axioms in the form of equations which relate the operations of type to each other [Gut78]. Mostly two specification methods are used: algebraic specification and operational specification [Loe87]. An operational method of specification is close to realization. A specific programming language is used for ADT specification. The set of values of ADT are constructed with the help of data structures of the programming language. The operations are defined with the help of the programming language. A defect of the operational specification method lies in its lack of abstraction. This method constitutes an implementation rather than specification. ADT represents mathematical models with associated methods which should be implemented by the term of ‘black boxes’ [HM08]. This method uses the axiomatic system and, therefore, it is independent from the implementation [Gut78] [GH78]. Heterogeneous algebra is used for the formal description of ADT [EM85] [LG86]. Heterogeneous algebra includes values of several types. This feature is appropriate for representing data types used in programming. According to the ADT theory only one data type is picked out and it is described by other types which were defined before [GH78]. The algebraic specification is more suitable for most authors to describe ADT.

It is recommended to group the functions of a descriptive data type for creating systematic axioms, i. e. generating, transforming, and reflecting. Sometimes the functions which add some feature to the existing value can be accepted as generating. The reflecting function which generates a new value of some ADT from the value of another data type can be accepted as generating as well.

A new data type can be specified systematically based on the tables of function results. Axioms must be written for every transforming and reflecting function. The number of axioms depends on the number of generating functions with the values suitable for a definable (transforming or reflecting) function (Fig. 3).

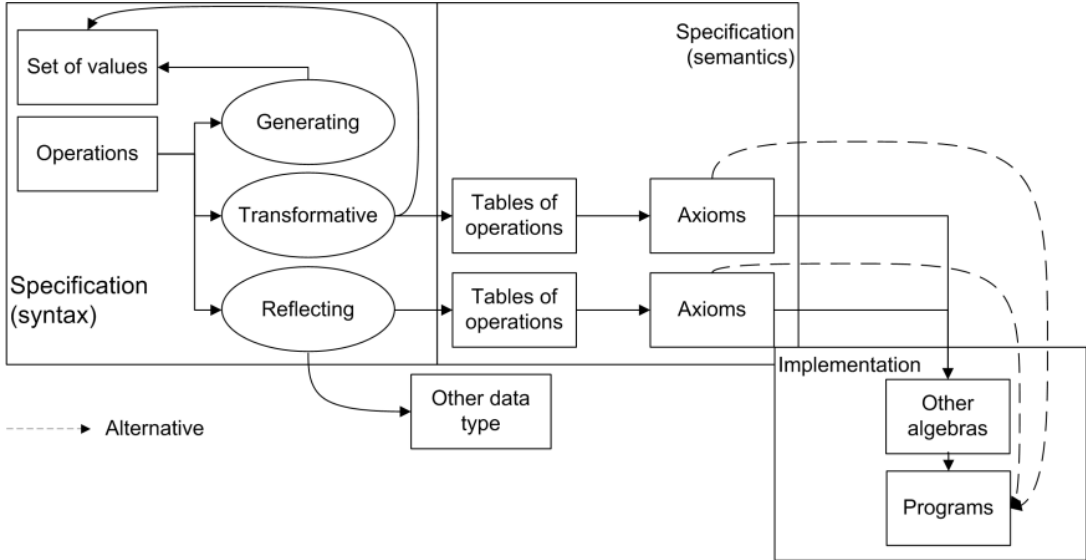


Figure 3. Systematic method for defining ADT

Dynamic geometry can be described as an application where the microworld of geometry is constructed using the interaction of different ADTs. Each microworld  $M_k$  is a sketch of dynamic geometry. The particularity of dynamic geometry requires that we describe scenarios step by step of each  $M_k$ . The method of heterogeneous algebra will be used to formally describe each scenario.

## 2.6. Conclusions

The analysis of literature proves that the usage of dynamic geometry systems reaches most of the goals of constructionist learning of mathematics. The research in mathematics learning by using dynamic geometry system validates the effectiveness of learning mathematics in various aspects: improve abilities, imagination, logical and problem thinking, etc. The studies reveal that teachers rarely use dynamic geometry. Some reasons were recognized why teachers did not use such a powerful tool: dynamic geometry systems are too complicated for teachers to use, traditional teaching is easier etc. It results in the following research question:

*how to propose to the teacher to use dynamic geometry?*

The answer to this question was accounted for in the descriptions of the information visualization and peculiarity of usage of dynamic geometry. A decision was made to help teachers to use dynamic geometry by presenting them created interactive microworlds. The review of scientific literature leads to the question:

*how to visualize geometry?*

The direction of our studies leads to the scientific problem:

*how to create an interactive geometry visualization model for effective teaching and learning?*

The relation between dynamic geometry and data structure was identified. The informatics method was found to solve the problem. The realization of microworlds by using *abstract data type theory* was developed.

## 3. The interactive visualization model

In this chapter, the interactive visualization model based on abstract data type model, constructionist learning model, and interactive visualization criteria model is presented and analysed. The ADTs are formalized by heterogeneous algebra. All specified ADT are based on the technological possibilities of dynamic geometry systems.

### 3.1. The model of constructionist learning of geometry

A few similarities can be observed in the development of game and to creation of dynamic sketches. The *Baytak* constructionist model approaches learning to create drawings with dynamic geometry. When learning to use dynamic geometry, the design step is changed by developing a drawing scenario, and testing is changed by drawings. Other steps remain the same. The grey part outside the schema (Fig. 4) presents the *Baytak* model, while the black part is an extended model adapted to geometry learning by using dynamic geometry.

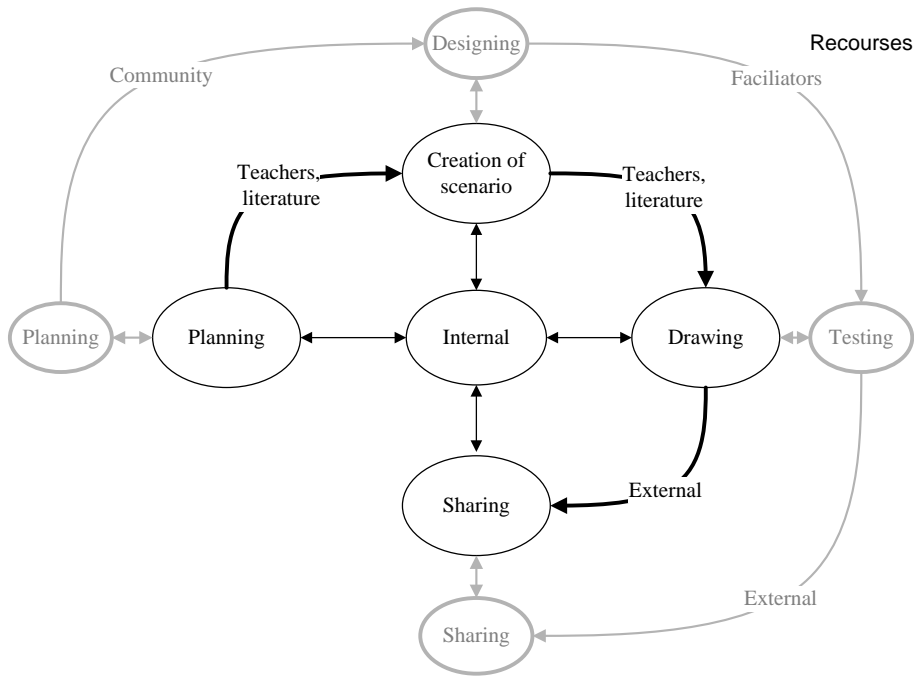


Figure 4. The extended and adapted *Baytak* model of constructionist learning with DGS.

The extended description of this model can be found in chapter 3.2 of the present dissertation.

The Kolb's [Kol05] experiential learning cycle is used in the research to explain the view of constructionist learning of mathematics by using dynamic geometry. The Kolb's learning cycle is implemented mainly for experiential purposes. Learning by dynamic geometry is based on students' experiences and practice.

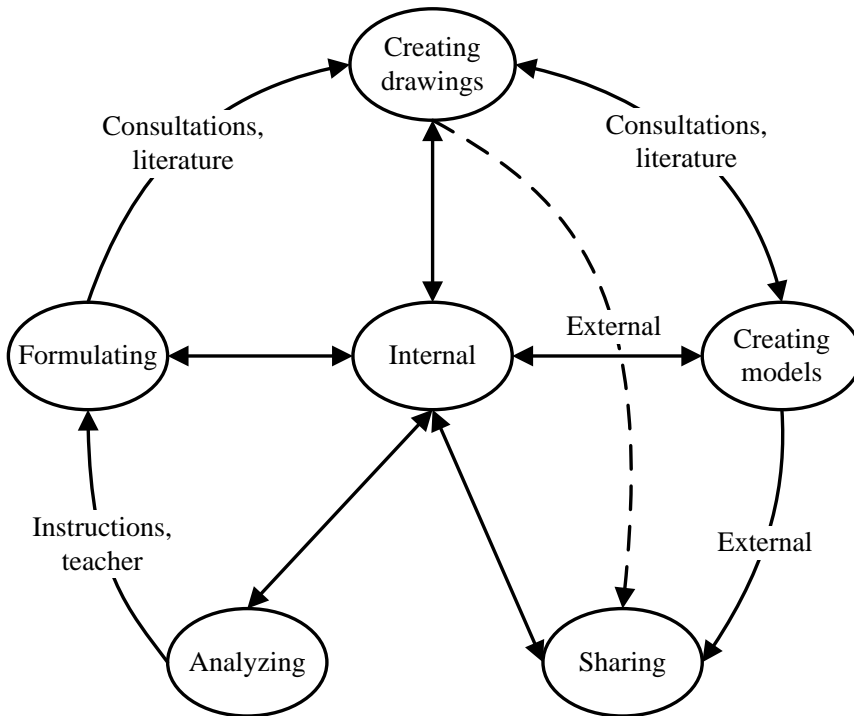


Figure 5. Extended constructionist learning model

In each classroom students differ in their learning styles. Thus, a teacher has to prepare learning material to meet all those learning styles in a lesson. Therefore, we are interested in Kolb's cycle of learning stages of mathematics and seek to make the learning process more effective and involve most of the students in learning. To this end, we have extended the model of geometry related to Kolb's cycle of learning stages (Fig. 5).

The model illustrates five steps of learning: 1) analyzing; 2) formulating; 3) creating a drawing; 4) a student shares and discusses his drawing with other students; 5) creating models which can be discussed with others. In all the steps a student constructs his own internal knowledge by interacting with the external learning environment (i. e. teachers, students, literature, etc.).

The developed model can be useful for teachers to prepare lessons. It helps to analyse the learning or teaching material and to find out how to adapt it to different learners and use it for constructionist mathematics learning

### **3.2. The requirements for an interactive visualization model**

Three requirements are applied for an interactive visualization model:

- 1) The model has to meet technological and user criteria according to ISO standards.
- 2) The model has to be adapted to any dynamic geometry system.
- 3) The model has to be related to mathematics curricula of Lithuania.

### **3.3. Criteria of interactive visualization**

The adaption and appliance of qualitative criteria in the earlier stages of artefact development help to develop a product of better quality [ISO09]. Therefore, the model of qualitative criteria is developed to satisfy creation of interactive geometry visualization model based on dynamic geometry system (Fig.7).

The final product of the implementation of the interactive visualization model is the collection of interactive microworlds, i. e. a digital learning tool. The qualitative criteria model is based on Standard of Systems and software of Quality Requirements and Evaluation [ISO12] and Casselmans requirements [Cas00] for static drawing. The model of technological and user criteria are presented in diagram of Figure 6.

The criteria are divided into two groups: criteria of interactive microworld and criteria of dynamic drawing.

The technological criteria for both products are the following:

- 1) Criteria of microworld: maintainability (modularity, modifiability), usability (user error protection, user interface aesthetics, easy to use), reliability (maturity, availability, fault tolerance, recoverability, accuracy), functional suitability, interactivity.
- 2) Criteria of dynamic maintainability (modifiability), usability (user error protection, user interface aesthetics, easy to use), reliability (maturity, availability, fault tolerance, recoverability, accuracy), functional suitability, interactivity.

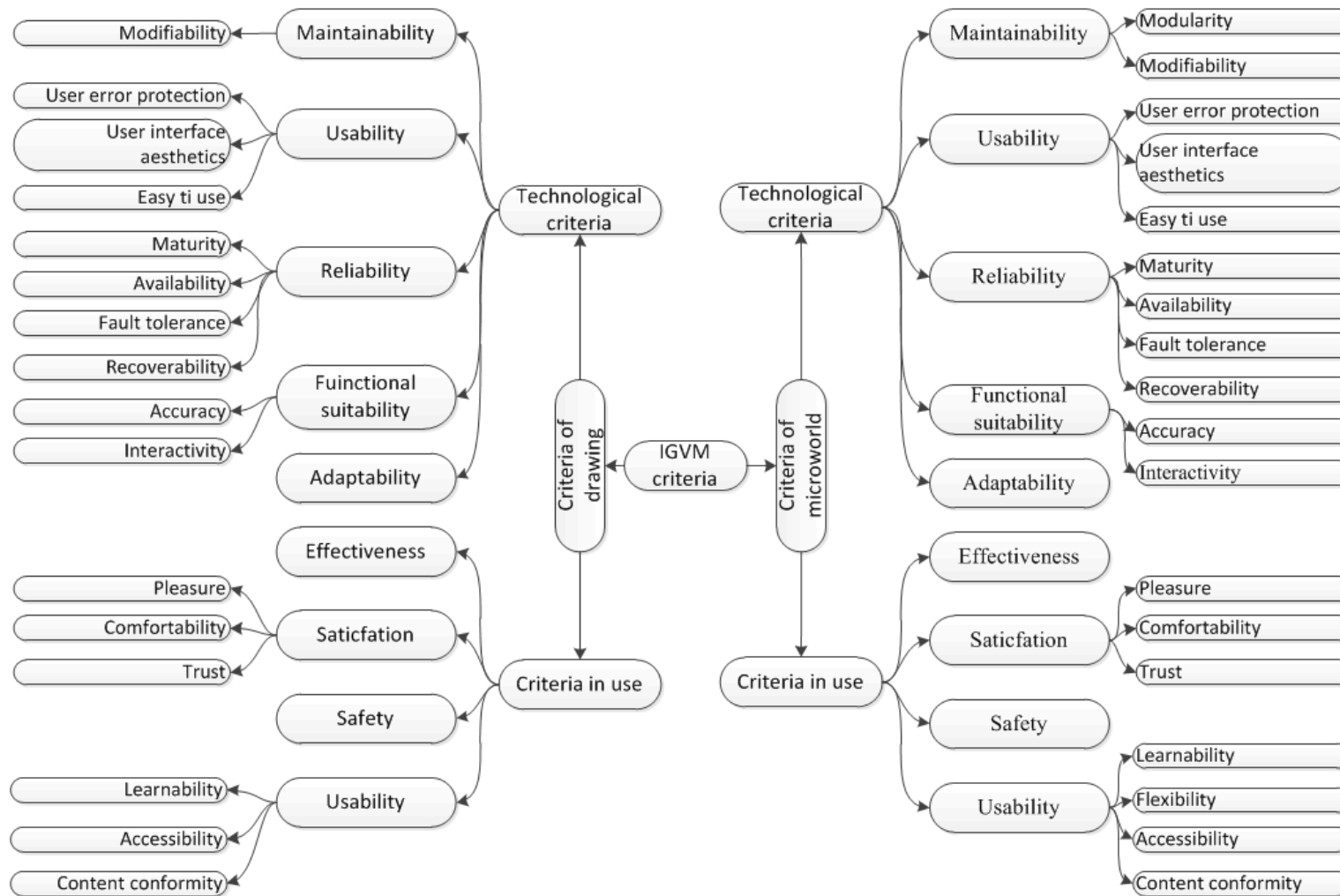


Figure 6. The model of qualitative criteria for interactive visualization

User criteria are also based on Standard of Systems and software Quality Requirements and Evaluation [ISO12].

- 1) Criteria of microworld: effectiveness, satisfaction (pleasure, comfortability, trust), safety, usability (learnability, flexibility, accessibility, content conformity).
- 2) Criteria of dynamic drawing: effectiveness, satisfaction (pleasure, comfortability, trust), safety, usability (learnability, flexibility, accessibility, content conformity).

This model of quality criteria is used for creating interactive microworlds, but it can be used to evaluate any digital learning tool.

### 3.4. The conceptual model of interactive visualization

The developed model of interactive geometry visualizations has three main stages (Fig. 7):

1) *the system of templates*. In this stage, the curriculum of mathematics is studied and the topics of interactive visualization are chosen and the list of interactive microworld has to be drawn. The templates are developed taking into account teaching and learning methods, criteria of visualization and technological possibilities.

2) *Formalization and scenarios*. The ADT theory is chosen to perform formalization. The scenarios are based on templates and described according to ADT syntax and semantics are written in this stage.

3) *Realization*. The written scenarios are implemented by use of technology (in this case the dynamic geometry system), the microworlds are created and joined into a digital learning tool.

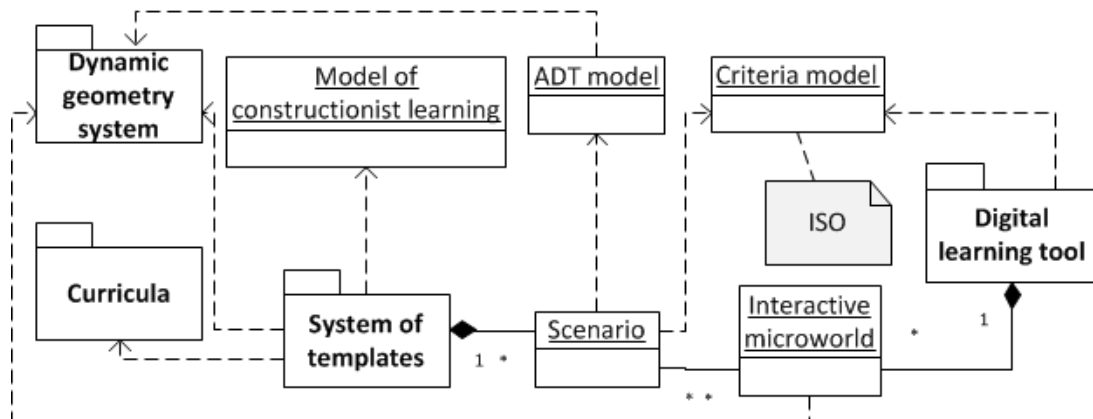


Figure 7. The model of interactive visualization

### 3.5. The diagram of activity of interactive visualization

A digital learning tool can be created by three steps: 1) analyze curricula, choose topics for visualization and develop templates, 2) write scenarios for each topic, 3) create a set of learning microworlds [DJ06]. Two different domains have merged here, i. e. technology and education of mathematics. The competencies of two different domains are required to create learning resources. The presented three steps have to be expanded in detail. The competence of education of mathematics allows review mathematics curricula and select topics for visualization. Technological sophistication allows to introduce features of technology (in this case dynamic geometry) and to prepare



templates of scenarios. The creation of scenarios requires pedagogical and mathematical knowledge. Afterwards the scenario has to be implemented in dynamic geometry. The interactive microworld of specific learning scenario is developed after its implementation. This microworld has to be tested by the actor of the scenario and improved if necessary. All these activities can be organized by a publisher (Fig. 8).

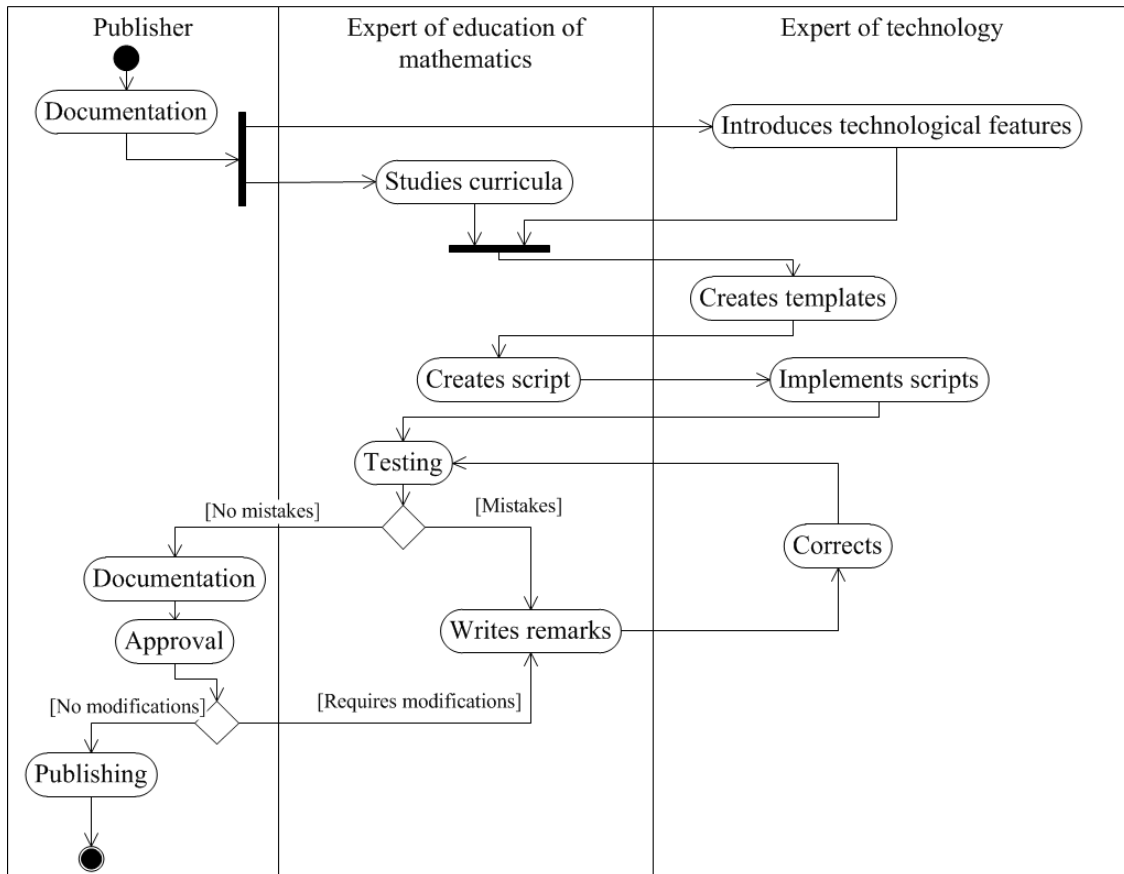


Figure 8. The activity diagram of interactive visualization

### 3.6. The user case diagram of interactive visualization

The model needs to be tested at least by three types of users: a technical expert, teacher and student. In order to create microworlds specific competencies have to be acquired: 1) management of dynamic geometry system for creation of interactive microworlds, and 2) pedagogical practice of teaching mathematics in class. Therefore, a technological expert is needed to implement scenarios by using dynamic geometry system.

The learning activities are presented in European Schoolnet Vocabulary Bank for Education [VBE]. The activities which correspond were picked out from the following vocabulary: create, learn, observe, practice, read, simulate and train. All activities by the user are presented in the use case diagram (Fig. 9)

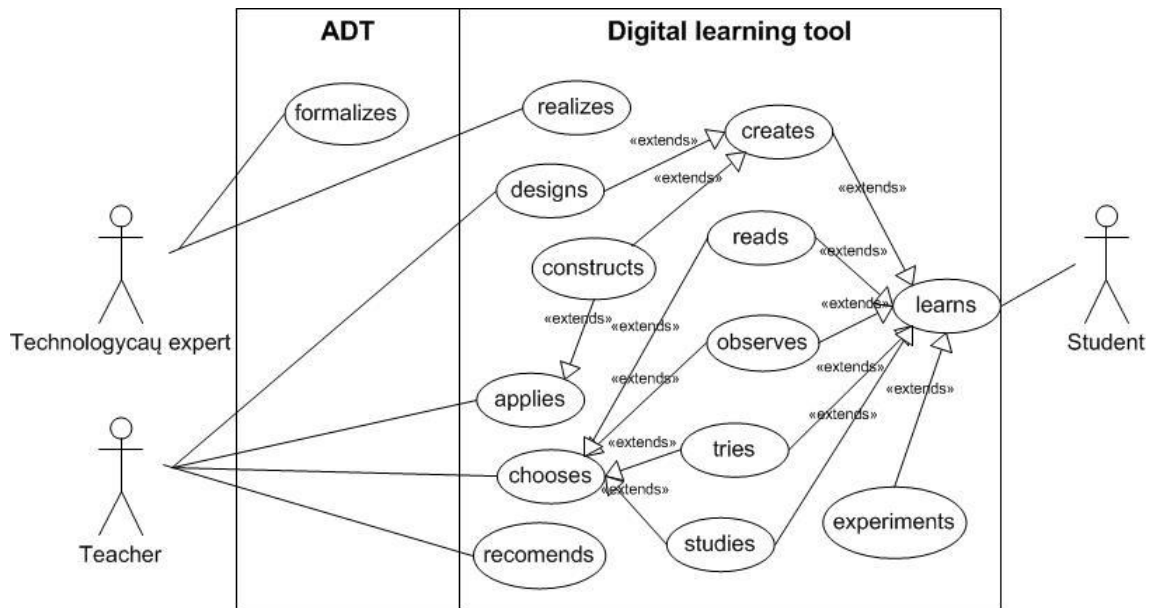


Figure 9. The case diagram of the product

### 3.7. Classification of templates for microworlds

The study of National curricula of mathematics of primary, basic and secondary education revealed classes of templates which can help to prepare microworlds of learning and teaching geometry [PPU08] [VUB08]. The class of scenario templates has such attributes as text, dynamic drawing, start positions, and action buttons. The scenario of classes of templates consists of four (Fig. 10):

- 1) Concept presentation. This template includes five attributes: dynamic drawing, text, action button, start position and animation.
- 2) Theorem and axiom presentation. The goal of the microworld is to discover and ensure the correctness of the theorem or axiom. Therefore, in most microworlds the new attribute measurement appears.
- 3) Property presentation. This class has two more attributes, i. e. expression and examples. The examples show the properties of separate ‘handy’ geometric objects.
- 4) Proof and method presentation. The constructions of the microworld class of this arc completely different from the previous classes. Although the attributes are very similar. Their implementation is more complex. An additional attribute – step appears, but it changes all the construction.

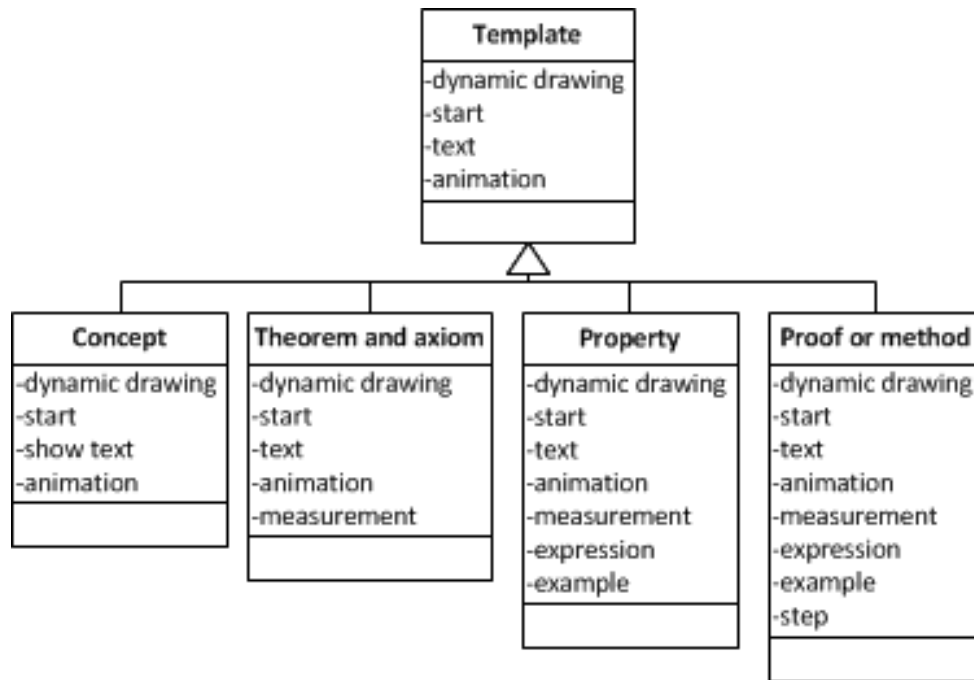


Figure 10. The classes of templates

### 3.8. ADT implementation in interactive geometry scenarios

The main ADT in the dynamic geometry sketch is `din.geom`. It will be defined when all ADTs are described. The centre of an interactive microworld is a dynamic drawing which consists of geometric objects. The `geom.obj` is the first of the type and it can be described by homogeneous algebra, but it is necessary to relate it with other definable types. Therefore, it is defined in the notation of heterogeneous algebra. The set of values of this data type is infinite and all its values are created by generating functions. All the values can be produced by a finite set of functions (Fig. 11). Functions are grouped into generating functions F1, transforming and reflecting functions F2 (Fig. 11).

Some of functions F1 do not create new values. They assign features to the existing values: `select`, `hide`, `t_set.mirror`, `t_set.centre`.

This type has only three transforming functions: `undo`, `op_point.obj` and `op_intersection.point`. The axioms for these transforming functions are presented below (see Dissertation).

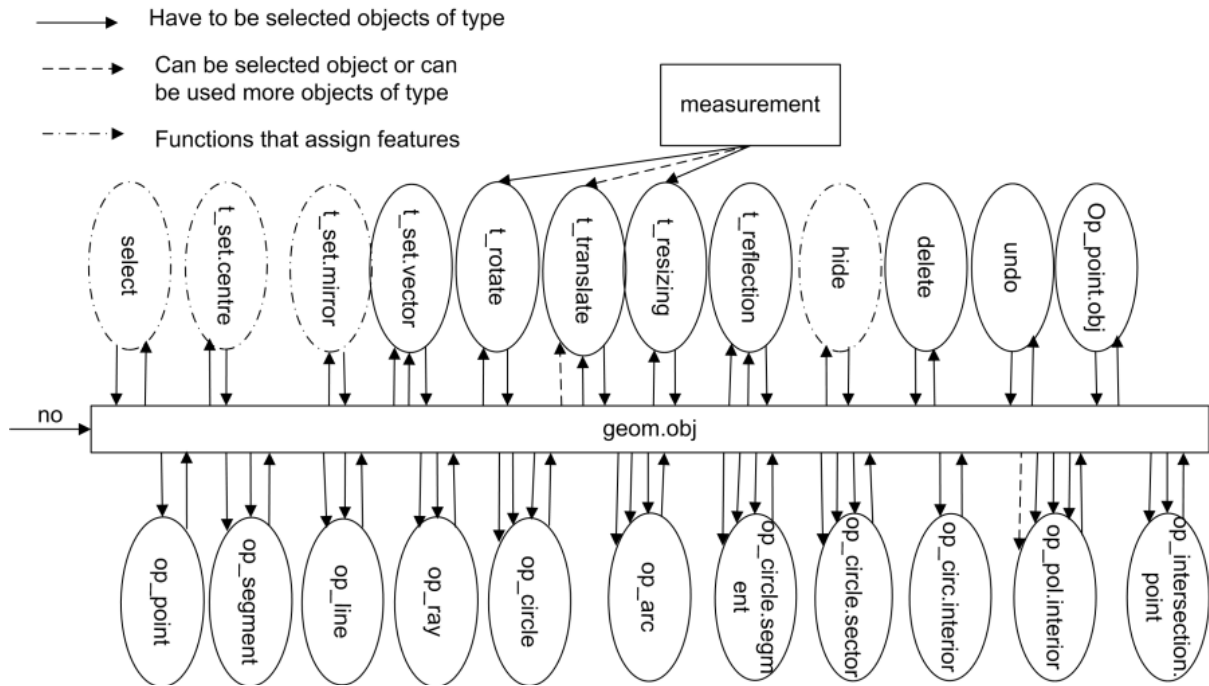


Figure 11. Graphical visualization of ADT `geom.obj`

Another ADT for a dynamic geometry is termed `measurement`. Specification of this type requires a method of heterogeneous algebra. Its set of values is infinite. All functions of real numbers are suitable for this type. Functions of this type are distributed in two groups, i. e. generating functions F3, transforming and reflecting functions F4. Reflecting functions reflect values of the data type `geom.obj` (Fig. 12).

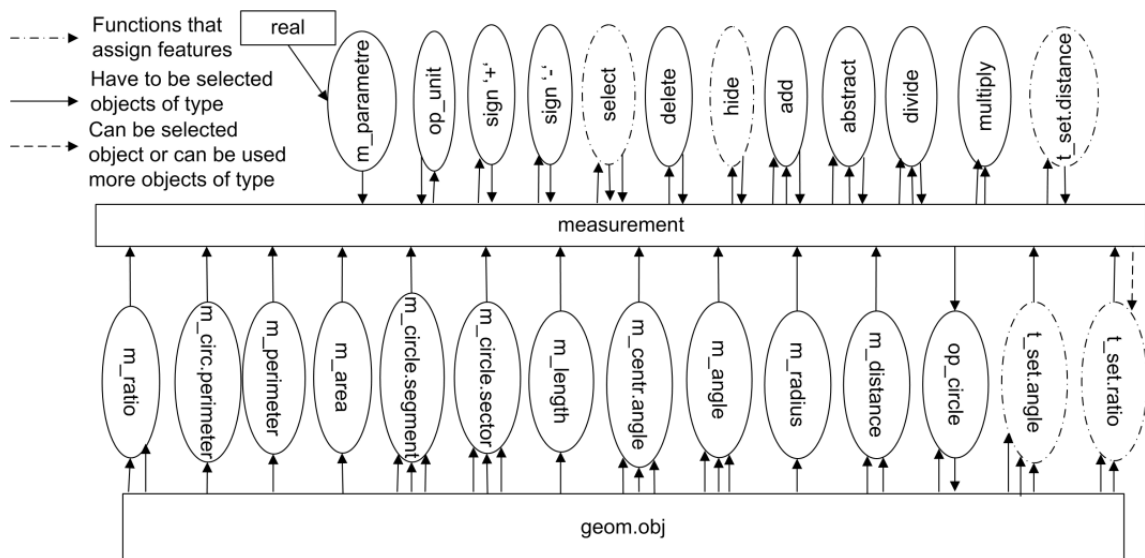


Figure 12. Graphical visualization of relations between ADTs `geom.obj` and `measurement`

The generating function `t_set.distance` assigns a feature to measurements. The reflecting functions `t_set.angle` and `t_set.ratio` assign features to the existing values. Axioms are constructed for transforming functions (see Dissertation).

The third type of ADT defined is `text.block`. The set of values of this type is infinite and the values are text blocks. Every block of this type consists of characters. To this end the data type `char` with generating functions `c_add`, `c_delete`, and `c_select` are used (Fig. 4). This type has a finite set of functions: generating functions F5 and reflecting functions F6 (Fig. 13).

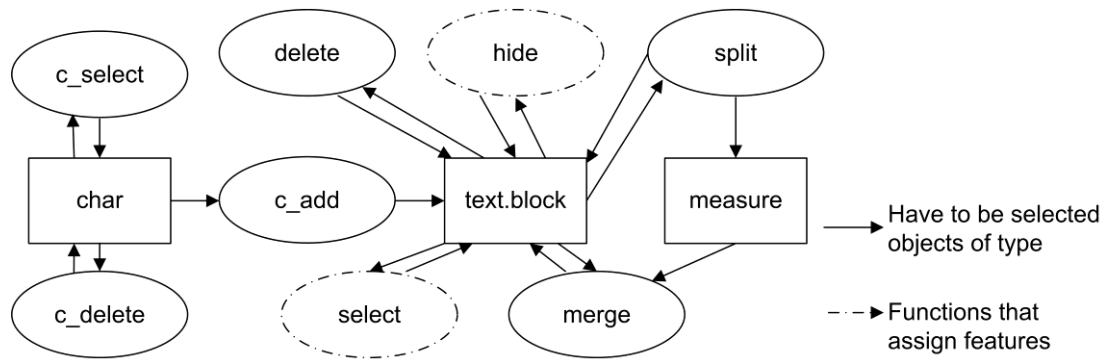


Figure 13. Graphical visualization of the type `text.block`

The last created ADT is `action.button`. The set of values of this type is finite. All values are generated by the reflecting functions (Fig. 14).

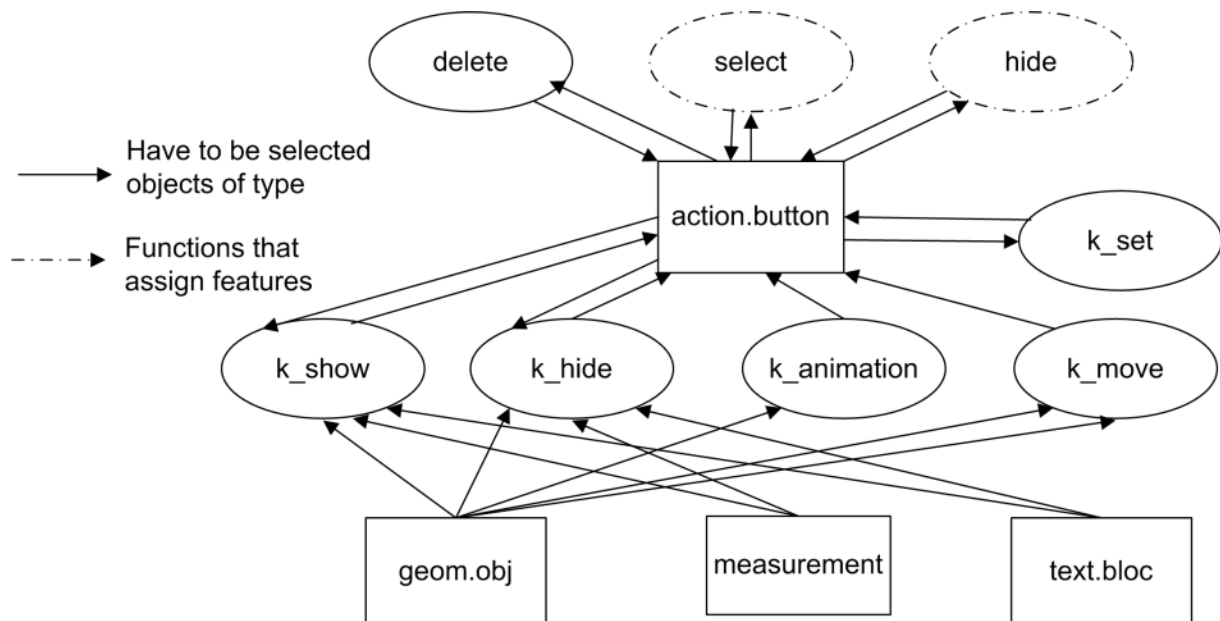


Figure 14. Graphical visualization of ADT `action.button`

All the described ADTs are described in the notation of heterogeneous algebra. In this way, the new ADT named `din.geom` was obtained. A set of values of this type is  $R = \{R1, R2, R3, R4\}$ , where set  $R1$  consists of values of the type `geom.obj`, set  $R2$  consists of values of the type `measurement`, set  $R3$  consists of values of the type `text.block` and set  $R4$  consists of values of the type `action.button`.

All the functions described for each data type are suitable for ADT:  $F = \{F1, F2, F3, F4, F5, F6, F7, F8\}$  (Fig. 15).

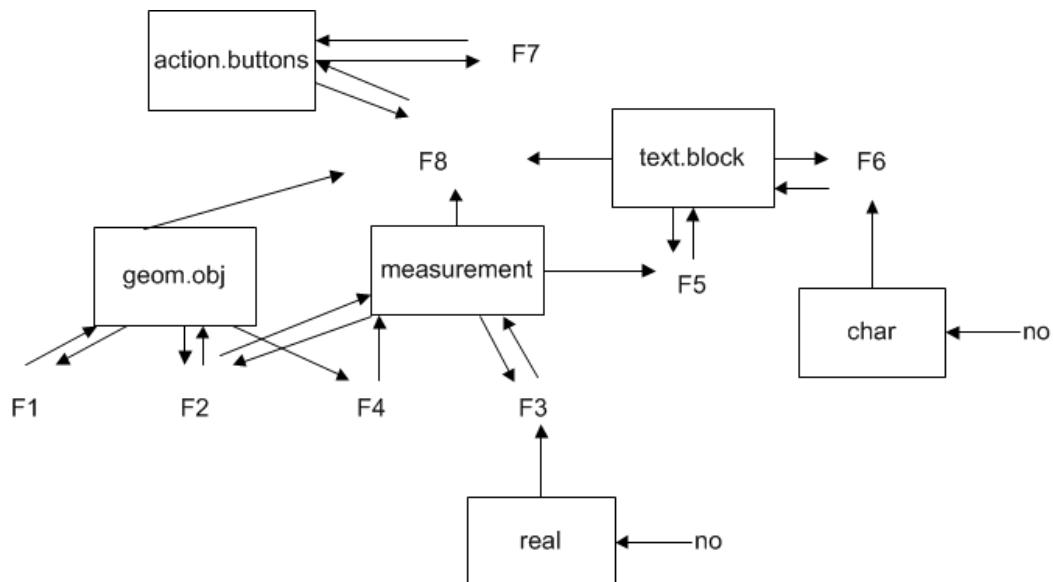


Figure 15. Schema of the ADT syntax in a dynamic geometry

## 4. The model implementation and validation

### 4.1. Introduction

The validation of a developed model has to be performed in the following steps: 1) the model implementation in four most popular dynamic geometry systems has to be shown; 2) one dynamic system has to be selected from all four alternatives of model implementation; 3) one topic from the curricula of mathematics has to be chosen for the implementation; 4) quasi experiment has to be performed.

### 4.2. The ADT implementation in four dynamic geometry systems

In this chapter, the model developed on ADT `din.geom` syntax and semantics is universal and it can be realized practically within any dynamic geometry system. There are more than 40 dynamic geometry systems described in scientific literature [Mac11] [AEB10] [HHL09] [LAS08] [JQ07]. Those various scientific databases were reviewed and experiment indicates that four of the systems are most popular: ‘Geometer’s sketchpad’ (described in 671 papers), ‘Geogebra’ (described in 284 papers), ‘Cabri’ (described in 1395 papers) ir ‘Cinderella’ (described in 202 papers).

To implement the ADT `din.geom` the following systems were used: ‘Geometer’s Sketchpad’ version 4, its licence allowed us to use it for learning and teaching purposes in all Lithuanian schools for free; ‘Geogebra’ 4.4; ‘Cabri II Plus’ 1.4 and ‘Cinderella’ 2.

A concept of implementation was chosen from the curricula of mathematics: ‘Area of triangle when two sides and the angle included are given’

The scenario of the microworld based on ADTT `din.geom` is described below:

Name: `area.trinagle.trig`

1. `op_point(no) = A1 {geom.obj→geom.obj}`
2. `op_point(no) = B1`
3. `op_point(no) = C1`
4. `op_segment(A1, B1) = segment_AB1`

5.  $op\_segment(B^1, C^1) = segment\_BC^1$
6.  $op\_segment(A^1, C^1) = segment\_AC^1$
7.  $angle.arc(A^1, B^1, C^1) = arc\_EF \{script\}$
8.  $angle.arc(B^1, A^1, C^1) = arc\_EF \{script\}$
9.  $angle.arc(A^1, C^1, B^1) = arc\_EF \{script\}$
10.  $m\_segment(AB^1) = a \{geom.obj \rightarrow measurement\}$
11.  $m\_segment(BC^1) = b$
12.  $m\_segment(AC^1) = c$
13.  $m\_angle(A^1, B^1, C^1) = \beta$
14.  $m\_angle(A^1, C^1, B^1) = \gamma$
15.  $m\_angle(C^1, A^1, B^1) = \alpha$
16.  $c\_add(empty) = t1 \{char \rightarrow text.block\}$
17.  $c\_add(empty) = t2 \{char \rightarrow text.block\}$
18.  $c\_add(empty) = t3 \{char \rightarrow text.block\}$
19.  $c\_add(empty) = t4 \{char \rightarrow text.block\}$
20.  $m\_area(op\_pol.interior(A_3^1)) = S \{geom.obj \rightarrow measurement\}$
21.  $merge(t1, b, t2, c, t3, \alpha, t4, S) = t7 \{text.block \rightarrow text.block\}$
22.  $merge(t1, a, t2, b, t3, \gamma, t4, S) = t7$
23.  $merge(t1, a, t2, c, t3, \beta, t4, S) = t7$
24.  $c\_add(empty) = t8 \{char \rightarrow text.block\}$
25.  $c\_add(empty) = t9 \{char \rightarrow text.block\}$

This scenario was performed in all the chosen systems. All steps were successfully implemented in all four systems. Some of the system had no relevant features. They were created with additional steps (scenarios). The scenarios and images of the implemented microworlds of all the systems are described in the chapter 4.2. of dissertation.

### 4.3. The comparison and selection of dynamic geometry system

One of the systems was chosen to evaluate the model's implementation during the experiment. To evaluate it the analytic hierarchy process (AHP), described by Saaty [Saa80] was used.

The alternatives were compared by the system features which allowed establish interactive visualization criteria. The criteria of pair comparison were chosen from the model of the interactive visualization criteria. The model of criteria and subcriteria was developed. The stress of criteria was compared in pairs and the weights were calculated by T. L. Saaty's method [Saa94]:

$$W = (0,0491; 0,1075; 0,05910; 0,1378; 0,0419; 0,0879; 0,1847; 0,1552; 0,0884; 0,0884)$$

The systems were compared in pairs based on specific criteria. The results of comparison are shown in Table 1 below. Two dynamic geometry systems are suitable for the implementation of interactive visualization model: ‘Geometer’s Sketchpad’, and ‘Geogebra’. The results are very similar of both systems.

Table 1. The evaluation of systems

	Evaluation
„Geometer’s Sketchpad“	0,592
„Geogebra“	0,582
„Cabri“	0,188
„Cinrerella“	0,172

The ‘Geometer’s Sketchpad’ was chosen for the implementation because of such subjective reasons as: 1) licence to use this system for free that Lithuanian schools have, and 2) use of ‘Geometer’s Sketchpad’ in Lithuania for ten years.

#### 4.4. Experiment

The efficiency of dynamic geometry for teaching and learning is based on several ways. The most relevant among them are: 1) the usage of dynamic geometry for teaching (and learning) geometric proofs [Jon00] [HJ98], 2) the usage of dynamic geometry for teaching (and learning) geometric concepts [GV10]. However, the most common use of dynamic geometry is the construction of sketches that the students make on their own when starting from pre-set instructions presented by the teachers. Such an effective learning method is undoubtedly interesting, but needs to be emphasized that many teachers find very complicated to use dynamic geometry or information technology in lesson of mathematics [SK11]. Therefore, we prepared the set of interactive images of dynamic geometry and its influence to the student’s geometric abilities as ascertained by the experiment. The following research questions of experiment are hence formulated:

- 1) Do interactive images created by the above mentioned model and used for geometry teaching have any effects on students’ geometric skills?
- 2) To what changes the interactive images lead in students’ geometric skills?
- 3) What skills are improved when interactive images are used for teaching-learning?

For this purpose the experiment on the usage of pre-created sketches in the learning mathematics has been conducted.

#### 3.2 Research design

Two teaching and learning methods for measuring of the experimental factors (independent variables) were used in the experiment: 1) interactive geometry images used for individual learning in computer class and 2) the interactive images used to visualize a teacher’s explanation.

In order to obtain more accurate results of the experiment, a sample for the experiment has been chosen according to several criteria: 1) all the experiment participants in the have to be taught by the same teacher, 2) the participants should not



have been taught previously mathematics using dynamic geometry, 3) all the participants have to belong to the same age group, 4) each group of participants must have similar ratio of girls and boys, 5) the mathematical competencies of all the participants has to be similar. Depending on this the experiment has been performed with 13-14 year students (grade 7). The whole sample has been distributed into two groups depending on the experimental factors:

- The experimental group E. It was divided into two groups: experiment of group E1, where the students were taught in a computer classroom. Each student had received tasks and instructions how to use pre-created interactive images individually. The experimental group E2, where the teacher used interactive images to demonstrate geometric topics during explanation for this class;
- The control group C. No interactive learning images or tools were used in this students' group.

The geometric topic 'Lines and angles' were chosen for the experiment. It was the first geometric topic to be taught in that school year to the students involved in the experiment. The geometric concepts, properties, and drawing methods of this topic were chosen for visualization by using dynamic geometry. Corresponding 16 interactive images were created with 'Geometer's Sketchpad'.

The dynamic sketches were created according to the above mentioned model and criteria of the interactive visualization. The teacher provided for group E1 worksheets with topic related questions and tasks while for group E2 prepared sketches for visualization purposes (text was increased; comments irrelevant to the demonstration were removed, etc.).

The study also involved a pre-test and post-test assessment for measurement. The results of pre-test and post-test of each group were compared (Fig. 17).

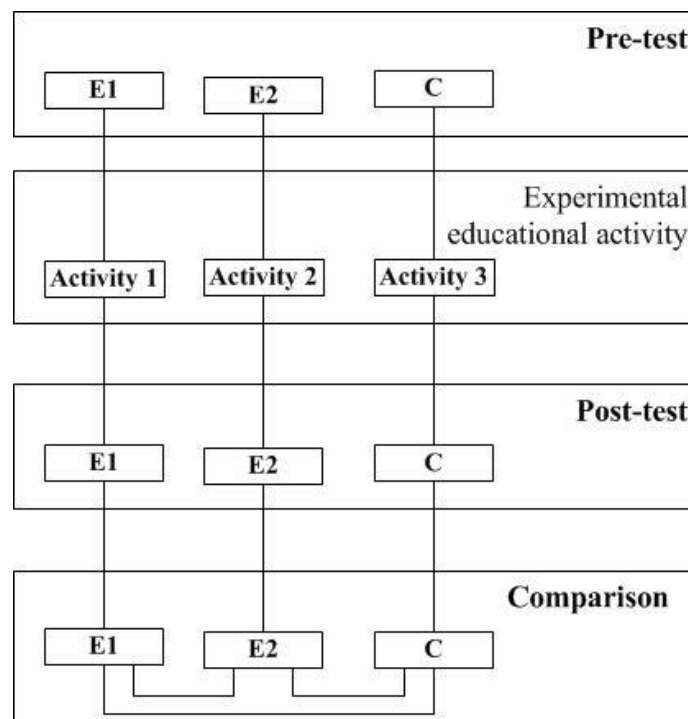


Figure 16. The model of the experimental research

The experiment has been conducted a month. Each group of the students participated in 14 mathematics lessons. The interactive images created by dynamic geometry were used in every second lesson of the experimental groups E1 and E2, whereas during other lessons they were taught without computers or interactive images.

T-test was used to compare average means of the experimental and control groups. The significance 0.05 was used, and the data were analyzed using SPSS program.

The results of each group pre-tests and post-tests show that the means of every group grow significantly (Table 2).

Table 2. The comparison of pre-and post-tests average means of the groups

		N	Average	Std deviation	t	p
E	Pre	44	27,909	8,653	6,033	0,0004
	Post	44	34,114	6,039		
E1	Pre	22	27,478	5,923	4,484	0,0001
	Post	22	32,739	5,643		
E2	Pre	22	28,696	10,581	4,231	0,00024
	Post	22	35,826	6,104		
K	Pre	23	33,261	5,216	2,286	0,03222
	Post	23	35,652	4,877		

We examined the mean growth to see if one group is reached significantly higher the others. Comparing growth of the experimental groups E1 and E2 with the control group we found out a significant difference between them, as illustrated in Table 4.

Table 3. The comparison of post-tests results of groups

	N	Average	Std deviation	Compare with	t	df	2p
E	44	6,205	6,822	C	2,590	65	0,012
E1	22	5,227	5,468	E2	0,948	42	0,358
E2	22	7,182	7,962	C	2,382	43	0,022
C	23	2,391	5,016	E1	1,810	43	0,077

On the whole, the usage of interactive images in class had significant influence on the students' skills. 2-tail significance factors show that using pre-created interactive images during demonstration by teacher improves student's skills significantly. Meaningful difference was also observed between the control group and the group who individually used interactive images (Table 3). The training process results of the control group significantly improved, thus indicating that traditional teaching methods cannot be completely rejected and abandoned.

Afterwards examined what skills were improved most when using the interactive images. The results of the experiment show that the experimental group E improved skills more than in the group C. The skills of identifying geometric objects and using geometric marks improved in all groups. The author of the dissertation supposes that the

fact that the pre-test has carried out with the students without any geometric skills in this school year had an effect on the results as well. The results of the post-tests show that simple geometric skills become clearer for all the participants after a few lessons. Figure 18 reveals which skills improved in group E more than in the control group. Additional research is required to understand why skills G4 and G7 did not improve in the experimental group with the significant mean growth.

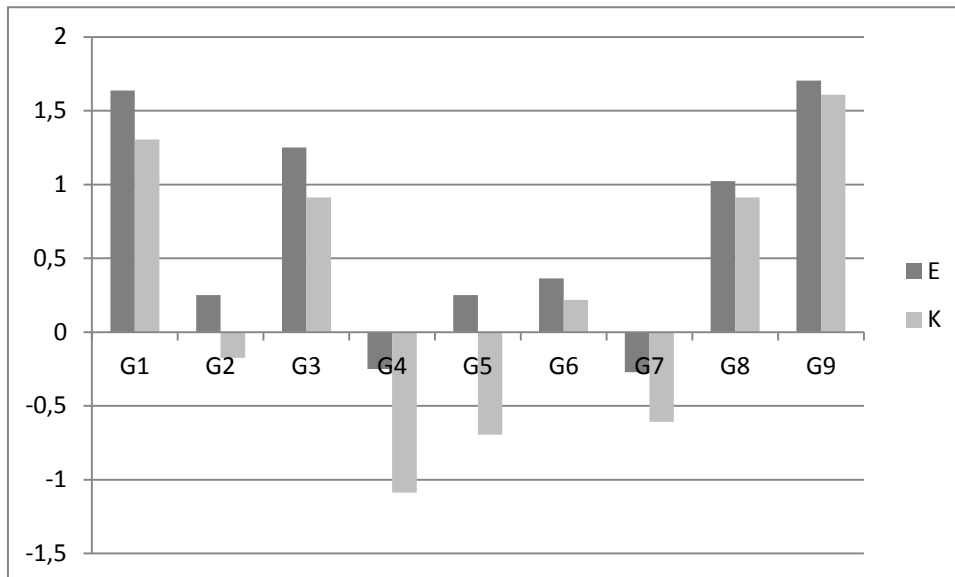


Figure 17. The comparison of students skills of E and C groups

#### 4.5. The substantiation of the interactive visualization model

The model of interactive visualization was successfully applied to visualize topics of mathematics in 9th and 10th grades. 190 microworlds were created to facilitate understanding of similar triangles, circle, trigonometric functions of acute angles, and exploration of triangles.

In addition, it was possible to visualize by the model of interactive visualization other non geometric topics such as: linear and quadratic functions, systems of linear equations, solutions of quadratic equations, graph of a function, set of equations and inequalities, and quadratic inequalities.

All sketches are recorder on CDs with descriptions how to use the sketches, theoretical material of a textbook, and recommendations on how to solve certain tasks in efficient way regarding textbooks of mathematics. More than 390 dynamic sketches were developed and CDs “Mathematics 9 with Geometer’s Sketchpad” and “Mathematics 10 with Geometer’s Sketchpad” were recorded and distributed throughout all Lithuanian schools.

#### General conclusions and results

1. The analyses of methods of development of digital learning tools allowed to chose and analyse theory of abstract data type (ADT) and specifications of heterogeneous algebra, to formalize semantics of dynamic geometry systems objects and operations by abstract data types which allows to describe scenarios of interactive visualization.
2. The model of interactive geometry visualization was developed and presented as a result of collected and systematized knowledge through the analysis of the methods

of informatics, features of dynamic geometry systems, sources about constructionist teaching (learning) and investigation of interactive visualization process. This model:

- 2.1. can help to create interactive microworlds based on the features of the chosen dynamic geometry system and the didactics of teaching and learning mathematics.
- 2.2. can be applied for interactive visualization of geometry and other topics of mathematic
3. The interactive visualization criteria model was created based on ISO/IEC standards. The proposed model:
  - 3.1. allows to observe the quality of digital learning tools in process of their creation and notice the errors in the early stage of designing the interactive microworlds.
  - 3.2. can be used to evaluate digital learning tools of mathematics.
4. The validation of the proposed model has proved that
  - 4.1. it can be implemented in various dynamic geometry systems;
  - 4.2. it is flexible, i.e. ADT model can be extended by new data types based on features of different dynamic geometry systems. The flexibility of the model allows to create interactive microworlds for teaching not only plane geometry but also functions, equations and systems of equations in algebra.
  - 4.3. the interactive microworlds created on the basis of the model of interactive visualization are effective and useful in education of mathematics. They improve most students' skills of mathematics.
5. The analysis of constructive and constructionist learning allows to expand and adapt *Baytak* model of constructionist learning of geometry by implementing interactive microworlds. The model allows to introduce dynamic geometry system into the education process of mathematics and offers guidance for teachers to effectively integrate interactive microworlds and dynamic geometry system in teaching their classes.

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### **Information about the author of the dissertation**

#### **Education**

1993-1999: gained master degree in mathematics education at Vilnius University.

2010-2014: doctoral studies in informatics at Vilnius University Institute of Mathematics and informatics

#### **Work experience:**

1997 till now: IT and mathematics teacher, Vilnius Jesuit gymnasium

2001-2005: engineer, Institute of Mathematics and Informatics

2005-2006: junior researcher, Institute of Mathematics and Informatics

2011-2014: project expert, VU Institute of Mathematics and Informatics

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## **Santrauka**

### **Darbo aktualumas**

Matematikos mokymas viena iš sudėtingiausių ir daugiausiai dėmesio reikalaujanti disciplina mokykliniame kurse. Mokytojo tikslas ne tik perteikti žinias, bet sudominti, motyvuoti ir įtraukti klasės mokinius į aktyvų mokymosi procesą konstruojant savo žinias ir gebėjimus.

Matematinių reiškinių vizualizavimas nuo pat Babilono laikų sudarė sąlygas šiems išlikti ir pasklisti. Šiuolaikinių dinaminių grafinių kompiuterių programų ir kitų mokymo technologijų atsiradimas itin stipriai veikia geometrijos vizualizavimą ir jos mokymą [Arc03] [Boz05] [GN07]. Atsiranda galimybės ne tik spręsti sudėtingus geometrinius uždavinius, bet ir pateikti juos aiškesne, suprantamesne forma. Vizualizuojant geometrinius konceptus, aksiomas ir teoremas galima lavinti mokinių vizualųjį mąstymą, vaizduotę, padėti susikaupti ties uždavinio sprendimu. Dėl to keičiasi matematikos edukologų požiūris į matematikos mokymą – informacinės technologijos tampa svaria sudėtine didaktikos dalimi [HHK08]. Vis daugiau dėmesio skiriama aktyviems mokymo metodams grįstiems konstrukcionizmo teorija, kurie skatina mokinius aktyviai veikti ir įsitraukti į mokomąją veiklą. Informacinės technologijos tampa viena efektyviausių priemonių padedančių įgyvendinti konstrukcionizmo idėjas ir įtraukiančių mokinius į aktyvią veiklą konstruojant matematinės žinias ir gebėjimus

Matematikos mokytojai dažniausiai linkę tikėti ir pabrėžia interaktyvios (dinaminės) geometrijos naudą mokinių mokymuisi, neabejoja, kad mokiniai gali geriau suprasti geometrinius konceptus, aksiomas, teoremas, ypač įrodymų konstravimą, tačiau patys prisipažįsta retai taikantys šias technologijas. Dažniausiai nurodoma kliūtis – nepakankami techniniai įgūdžiai, kurie neleidžia patiems efektyviai kurti naujų pavyzdžių. Dažnai matematikos mokytojai nemoka sparčiai naudotis dinaminės geometrijos sistemomis ir neturi laiko šiai kompetencijai įgyti. Šiai problemai išspręsti gali padėti mokyklinės geometrijos vizualizavimas panaudojant dinaminės geometrijos sistemas.

Todėl šiame tiriamajame darbe sukurtas interaktyvaus geometrijos vizualizavimo modelis paremtas dinaminės geometrijos sistema. Dinaminės geometrijos sistemą apibūdiname kaip programinę įrangą geometrijai mokyti, kurios pagrindinė savybė – dinaminių brėžinių konstravimas, analizė ir animacija (pavyzdžiui, „Geometer’s SketchPad“, „Geogebra“, „Cabri“).

### **Tyrimo objektas**

Informatikos metodų adaptavimas geometrijos interaktyviam vizualizavimui.

### **Tikslas**

Sudaryti interaktyvaus vizualizavimo modelį geometrijos konstrukcionistiniam mokymui ir mokymuisi, grįstą dinaminės geometrijos sistema.

### **Uždaviniai**

1. Ištirti informatikos metodų taikymą konstrukcionistiniam geometrijos mokymui.
2. Nustatyti geometrijos interaktyvaus vizualizavimo kriterijus.

3. Sukurti interaktyvaus geometrijos vizualizavimo modelį.
4. Pritaikyti abstrakčių duomenų tipų teorijos principus interaktyviojo vizualizavimo scenarijams aprašyti.
5. Realizuoti sudarytąjį modelį ir atlikti jo validavimą.

### **Ginamieji teiginiai**

1. Dinaminė geometrija – efektyvi priemonė, tinkanti geometrijai mokytis naudojant konstrukcionistinius metodus.
2. Sukurtas interaktyvaus vizualizavimo modelis geometrijai mokytis ir mokytis realizuojamas įvairiose dinaminės geometrijos sistemose.
3. Abstrakčių duomenų tipų teorija pritaikoma interaktyvaus vizualizavimo scenarijams aprašyti.
4. Pasiūlytas modelis padeda sukurti kokybišką skaitmeninę priemonę geometrijai mokytis.

### **Mokslinis naujumas**

Šio mokslinio darbo naujumą galima įžvelgti keliuose srityse:

1. *Informatikoje*: Pritaikyta abstrakčių duomenų tipų teorija interaktyvaus vizualizavimo dinaminės geometrijos sistemoje modelio formalizavimui ir realizavimui Formalizuota dinaminės geometrijos sistemos objektų ir operacijų sintaksė ir semantika abstrakčiais duomenų tipais. Sukurtas ADT modelis grįstas dinaminės geometrijos sistemoje naudojamomis priemonėmis ir sukuriama objektais.
2. *Informatikos inžinerijoje*: Sukurtas interaktyvaus vizualizavimo modelis geometrijai mokytis ir mokytis. Sudarytas specifikuotas kriterijų modelis skaitmeninės geometrijos mokymosi priemonės kūrimui.
3. *Edukologijoje*: Sudarytas konstrukcionistinio matematikos mokymo, naudojant dinaminės geometrijos sistemą, modelis.

### **Darbo rezultatų praktinė reikšmė**

Praktinė darbo reikšmė įžvelgiama keliais aspektais:

1. Formalizuota ADT sintaksė ir semantika, kurie ateityje galėtų padėti automatizuoti dinaminių brėžinių braižymą.
2. Išplėstas *Baytak* konstrukcionistinis mokymo(si) modelis suteikia gaires mokytojui, kaip efektyviai panaudoti dinamines geometrijos sistemas naudojant konstrukcionistinius metodus.
3. Sudarytas interaktyvaus vizualizavimo kokybės kriterijų modelis padedantis kokybiškai vizualizuoti matematikos kursą.
4. Interaktyvaus geometrijos vizualizavimo dinaminėje geometrijoje modelis pritaikytas vizualizuojant 9–10 klasių plokštumos geometrijos, funkcijų, tiesinių lygčių sistemų, tiesinių ir kvadratinių lygčių kursą.

### **Darbo apimtis ir struktūra**

Darbą sudaro: terminų ir santrumpų žodynas, 4 pagrindiniai skyriai, išvados ir

priedai. Apimtis be priedų – 137 puslapiai, panaudoti 35 paveikslėliai ir 10 lentelių, cituojami 134 literatūros šaltiniai.

## **Bendrosios išvados ir rezultatai**

1. Ištyrus informatikos metodus taikomus kuriant edukacines skaitmenines priemones pasirinkta ir išanalizuota abstrakčiųjų duomenų tipų (ADT) teorija ir jų specifikacija heterogeninėje algebroje; formalizuota dinaminės geometrijos objektų semantika abstrakčiais duomenų tipais, kuri gali padėti aprašyti scenarijus interaktyviam vizualizavimui.
2. Remiantis informatikos metodų analize, dinaminių geometrijos sistemų galimybių tyrimu, konstrukcionistinio mokymo(si) literatūros analize ir sukaupta interaktyvių vaizdų kūrimo patirtimi pasiūlytas interaktyvaus geometrijos vizualizavimo modelis, kuris
  - 2.1. leidžia kurti interaktyvius mikropasaulius atsižvelgiant į naudojamos dinaminės geometrijos sistemos technologines galimybes ir matematikos mokymo didaktiką;
  - 2.2. Leidžia kurti mikropasaulius ne tik geometrijos, bet ir kitų temų vizualizavimui.
3. Remiantis skaitmeninių mokymosi priemonių ir mokymosi objektų kokybės vertinimo standartais sudarytas ir pasiūlytas interaktyvaus vizualizavimo kokybės kriterijų modelis, kuris
  - 3.1. leidžia kontroliuoti interaktyvios skaitmeninės priemonės kokybę kūrimo procese, o tai leidžia pastebėti klaidas ankstyvoje mikropasaulių projektavimo stadijoje;
  - 3.2. gali būti panaudotas vertinant jau sukurtą skaitmeninę mokymo(si) priemonę geometrijos ir kitų matematikos temų mokymui ir mokymuisi.
4. Atliktas modelio validavimas leidžia teigti, kad pasiūlytas interaktyvaus geometrijos vizualizavimo modelis
  - 4.1. gali būti taikomas įvairiose dinaminės geometrijos sistemose;
  - 4.2. leidžia sukurti interaktyvius mikropasaulius ne tik plokštumos geometrijai mokyti, bet ir algebrinėms funkcijoms, lygtims, sistemoms.
5. Pagal interaktyvaus geometrijos vizualizavimo modelį sukurti interaktyvūs mikropasauliai lavina mokinių geometrinius gebėjimus mokantis savarankiškai ar naudojant interaktyvius mikropasaulius demonstravimui pamokoje.
6. Atlikus konstruktyvistinio ir konstrukcionistinio mokymo(si) teorijos analizę išplėstas ir adaptuotas Baytak modelis konstrukcionistiniam geometrijos mokymui ir mokymuisi, kuris padeda integruoti dinaminės geometrijos sistemą į edukacinį procesą; sukuria gaires mokytojui, kaip efektyviai panaudoti dinaminės geometrijos sistemą konstrukcionistiniam mokymui(si).